ABSTRACTS

FACTORIZATIONS OF ONE-GENERATED FORMATIONS

Clara Calvo
(in collaboration with A. Ballester-Bolinches and R. Esteban-Romero)

The concept of $\mathfrak{X}$-local formation, where $\mathfrak{X}$ is a class of simple groups with a completeness property, generalises the notions of local and Baer-local formation. It was introduced by P. Förster with the purpose of presenting a common extension of the Gaschütz-Lubeseder-Schmid and Baer theorems. Since the intersection of a family of $\mathfrak{X}$-local formations is again an $\mathfrak{X}$-local formation, there exists the smallest $\mathfrak{X}$-local formation containing a group G, that is, the $\mathfrak{X}$-local formation generated by G. Skiba posed in the Kourovka Notebook a question about Baer-local formations generated by a group. A more general version of this question is answered by using $\mathfrak{X}$-local formations. A complete description of the factorizations of the form $\mathfrak{F} \circ \mathfrak{G}$ of one-generated $\mathfrak{X}$-local formations is also given.

ON THE PROBABILITY OF MUTUALLY COMMUTING n-TUPLES IN COMPACT GROUPS

Ahmad Erfanian

Let G be a finite group. Then an ordered n-tuple $(x_1, x_2, \ldots, x_n)$ of elements in G with the property that $x_ix_j = x_jx_i$ for all $1 \leq i,\ j \leq n$ is called mutually commuting n-tuple. Now, let $\mathrm{Pr}_n\mathrm{Com}(G)$ denotes the probability that a randomly chosen ordered n-tuple of elements in G be a mutually commuting n-tuple. We aim to generalize the above concept to a compact topological group which generally not only finite but also even uncountable. We give some evaluations of $\mathrm{Pr}_n\mathrm{Com}(G)$ for non-abelian compact group G when $G/Z(G)$ is p-elementary abelian.

FINITE GROUPS WHOSE PROPER SUBGROUPS ARE p-SUPERSOLUBLE FOR A PRIME p

Ramon Esteban-Romero
(in collaboration with A. Ballester-Bolinches)

We say that a group G is *critical* for a class of groups $\mathfrak{X}$ or a *minimal non-$\mathfrak{X}$-group* whenever G does not belong to $\mathfrak{X}$, but all proper subgroups of G belong to $\mathfrak{X}$. A detailed knowledge of critical groups for a class of groups $\mathfrak{X}$ can give some insight into what makes a group belong to a class $\mathfrak{X}$.

Many mathematicians have studied critical groups for several classes of groups. For instance, Miller and Moreno (1903) and Rédei (1947) studied the minimal non-abelian groups, while Schmidt (1924), Gol'fand (1948), and Rédei (1956) have studied the critical groups for the class of all nilpotent groups. These groups are now known as Schmidt groups. Itô (1951) has studied the minimal non-p-nilpotent groups, for a prime p. They turn out to be minimal non-nilpotent-groups. Doerk (1966) and Nagrebeckiĭ (1975) have studied the minimal non-supersoluble groups.

A group G is *p-supersoluble* whenever it is p-soluble and all p-chief factors are cyclic (p a prime). Kontorovič and Nagrebeckiĭ (1975) classified the minimal non-p-supersoluble groups. Tuccillo (1992) attempted to classify the minimal non-p-supersoluble groups, but unfortunately there is a gap in his results which makes that some minimal non-p-supersoluble groups are missing in his classification.

In this talk we present a complete classification of the minimal non-p-supersoluble groups in the p-soluble universe. This corrects Tuccillo's gap. These groups turn out to fall into twelve types. As a corollary, the classification of minimal non-supersoluble groups follows.

A CONTRIBUTION TO THE THEORY OF FINITE SUPERSOLUBLE GROUPS, II

Luis M. Ezquerro
(in collaboration with A. Ballester-Bolinches and A. Skiba)

In this note we present some characterizations of the supersolubility of a finite group in terms of the partial cover-avoidance property of the subgroups of some relevant families of subgroups.

A STRUCTURE OF GALOIS GEOMETRICAL SPACE INTRINSICALLY CONNECTED WITH EACH FINITE GROUP OF EXPONENT 3

Domenico Lenzi

If $(G, \bullet)$ is a group of exponent p, the set $\sum_p$ of subgroups of order p determines a group partition of G, wiz. the subgroups in $\sum_p$ pairwise intersect in 1, moreover their set-theoretical union is G. Thus it is immediate to verify that if $\mathcal{L}$ is the set of the left cosets determined by the subgroups of order p ($\mathcal{L}$ coincides with the analogous set of right cosets), then two distinct elements of $\mathcal{L}$ intersect at most in one element of G; moreover, for any two distinct elements of G there is a (unique) element of $\mathcal{L}$ containing them. This means that $\mathcal{L}$ gives G a structure of line space $(G, \mathcal{L})$. Then one says that two elements of $\mathcal{L}$ are (left) parallel whenever they are the left cosets of a same subgroup of order p.
Nevertheless in the particular case in which $p = 3$ and G has order 3^n we define on $\mathcal{L}$ a different parallelism relation $//$. Namely we say that two lines of a plane $\prod$ of $(G, \mathcal{L})$, i.e. two elements of $\mathcal{L}$ contained in $\prod$, are parallel whenever they coincide or are disjoint. Then the following property holds:

i) $(G, \mathcal{L}, //)$ is the affine Galois space of order 3 (lines of 3 points) and dimension n, even if $(G, \bullet)$ is not commutative.
We demonstrate **i**) in two steps: in the first one we prove that the planes of $(G, \mathcal{L})$ possesse the structure of the affine Galois plane of order 3; in the second step we prove that $(G, \mathcal{L}, //)$ is a desarguesian space.
Then we have an obvious question: whenever $p \neq 3$, property **i**) can be transferred to some non-commutative finite group of exponent p?

THE GENERAL STRUCTURE OF PROJECTIVE PLANES OF ORDER n ADMITTING $PSL(2,q)$, $q > 3$, AS A COLLINEATION GROUP

Alessandro Montinaro

Projective planes of order n admitting $PSL(2,q)$, $q > 3$, as a collineation group are investigated for $n \leq q^2$. As a consequence, affine planes of order n admitting $PSL(2,q)$, $q > 3$, as a collineation group are classified for $n < q^2$ and $(n,q) \neq (5,16)$. A generalization of the Foulser-Johnson theorem is also obtained.

GROUPS WITH SMALL NON-NORMAL SUBGROUPS

Carmen Musella

The structure of groups in which every subgroup is normal is well kwown. Groups for which the set of non-normal subgroups is small in some sense have been studied by many authors in several different situations. In particular, in a a series of relevant papers, Romalis and Sesekin investigated the behaviour of soluble groups in which every non-normal subgroup is abelian. The aim of this paper is to study groups in which every subgroup either is normal or has finite commutator subgroup.

ON QUOTIENTS OF GROUPS HAVING A SERIES WITH FINITE FACTORS

Chiara Nicotera

For any class $\mathcal{X}$ of groups we denote by $\hat{P}\mathcal{X}$ the class of groups G having a series whose factors belong to $\mathcal{X}$.

A group G is locally $\mathcal{X}$ - graded if every nontrivial finitely generated subgroup of G has a nontrivial image that belongs to $\mathcal{X}$. If $\mathcal{X} = \mathcal{F}$, the class of finite groups, then locally $\mathcal{X}$ - graded groups are simply called locally graded groups and this class of groups frequently appears in literature mainly in the study of groups G that do not have infinite finitely generated simple groups as subgroups. If we denote by $\mathcal{L}(X)$ the class of locally $\mathcal{X}$ - graded groups, then it is easy to prove that $\hat{P}\mathcal{X} \subseteq \mathcal{L}(X)$ and Brodskii proved that if the class $\mathcal{X}$ is a quasi-variety then $\hat{P}\mathcal{X} = \mathcal{L}(X)$.

The inclusion $\mathcal{L}(X) \subseteq \hat{P}\mathcal{X}$ does not hold in general. For instance the direct limit A_∞ of finite alternating groups A_n for all positive integers n is a locally graded group that does not belong to $\hat{P}\mathcal{F}$. So $\hat{P}\mathcal{F} \neq \mathcal{L}(F)$.

The class $\hat{P}\mathcal{F}$ is not quotient closed, as well as the class $\mathcal{L}(F)$, for free groups belong to $\hat{P}\mathcal{F}$. P.Longobardi, M.Maj and H. Smith proved that if G is a locally graded group and H is a locally nilpotent normal subgroup of G, then the factor group G/H is locally graded. It is possible to prove that the same result holds also if we replace the class $\mathcal{F}$ by the class $\mathcal{A}$ of abelian groups and this implies that if $G \in \hat{P}\mathcal{F}$ and A is a locally nilpotent normal subgroup of G, then $G/A \in \hat{P}\mathcal{F}$.

ON INFINITE GROUPS WHOSE ASCENDANT SUBGROUPS ARE PERMUTABLE

T. Pedraza
(in collaboration with A. Ballester-Bolinches, L.A. Kurdachenko and J. Otal)

A subgroup H of a group G is said to be *permutable* (or *quasinormal) in* G if $HK = KH$ for every subgroup K of G. Like normality, permutability is not a transitive relation. This makes interesting the study of PT–groups, that is, groups in which permutability is a transitive relation in G. A well known theorem due to Ore shows that in finite groups every permutable subgroup is subnormal. Therefore, a finite group G is a PT–group if and only if each subnormal subgroup of G is permutable in G. The study of PT–groups begins with a paper of Zacher ([6]). Zacher determined the structure of finite soluble PT-groups in a similar way to Gaschütz's characterization of finite soluble groups in which normality is a transitive relation (i.e. finite soluble *T–groups*). More recently, Beidleman *et al.* ([4]) and Ballester-Bolinches *et al.* ([1]) have obtained local characterizations of finite soluble PT-groups in terms of their Sylow structure.

In general, every permutable subgroup is ascendant ([5]) although the converse is false. Therefore it is natural to consider groups whose ascendant subgroups are permutable. A group G is said to be an *AP–group* if every ascendant subgroup of G is permutable in G. Obviously, every AP–group is a PT–group because the relation *to be an ascendant subgroup* is transitive. Moreover, in finite groups these two concepts coincide. However, for infinite groups the classes of AP–groups and PT–groups are not the same. In this talk, we present some results about locally soluble hyperfinite AP–groups which show that the structure of these groups is very similar to the structure of finite soluble PT–groups ([2,3])

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ON THE RELATIVE COMMUTATIVITY DEGREE OF A SUBGROUP OF A FINITE GROUP

Rashid Rezaei

The aim of this article is to give a generalization of the concept of commutativity degree of a finite group G denoted by $d(G)$ which is the probability that a randomly chosen pair (x, y) of elements of G commute, to the concept of relative commutativity degree of a subgroup H of a group G (denoted by $d(H, G)$). We shall state some results concerning the new concept which are mostly new or improvements of known results given by Lescot in 1995 and Moghaddam in 2005. Moreover, we shall define the relative n-th nilpotency degree of a subgroup of a group and give some results concerning this at the end.

ON THE CENTRALIZER OF THE HYPERQUASICENTER OF A GROUP

Francesco Russo

A recent result of J. Beidleman and H. Heineken shows that the hyperquasicenter $Q^*(G)$ of a group G coincides with the largest hypercyclically embedded normal subgroup of G. Starting from this result, we investigate on the section $G/C_G(Q^*(G))$, where $C_G(Q^*(G))$ denotes the centralizer of $Q^*(G)$ in G. It is proved that such section is supersolvable under the maximal condition on the normal subgroups.

CENTRALIZER OF ENGEL ELEMENTS IN A GROUP

Carmela Sica
(in collaboration with Gérard Endimioni)

Let G be a group and H a subgroup. Given some information about the centralizer $C_G(H)$, what can we deduce about G? This type of problem is well-known and has been studied by a number of people. We are specially interested in the following result of Onishchuk and Zaĭtsev ([2]).

Proposition 1. *Let $\mathfrak{X}$ denote the class of polycyclic groups, the class of Černikov groups, the class of minimax groups, or the class of groups of finite rank (in the sense of Prüfer). Let G be a nilpotent group, H a finitely generated subgroup. Then $C_G(H)$ belongs to $\mathfrak{X}$ if and only if G belongs to $\mathfrak{X}$.*

In the paper [1], we aimed to obtain similar results, for instance when G is soluble. In particular, we strengthen the theorem of Onishchuk and Zaĭtsev when $\mathfrak{X}$ is the class of polycyclic groups or the class of Chernikov groups. If we write $L(G)$ for the set of left Engel elements of G, and $\overline{L}(G)$ for the set of bounded left Engel elements, we can state our main results as follows:

Theorem 2. *Let G be a soluble group. Let H be a finitely generated subgroup contained in $\overline{L}(G)$ and suppose that $C_G(H)$ is polycyclic. Then G is polycyclic.*

Theorem 3. *Let G be a radical group. Let H be a finitely generated subgroup contained in $L(G)$ and suppose that $C_G(H)$ is a Čhernikov group. Then G is a soluble Čhernikov group.*

References:

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ON THE ISOMORPHISM PROBLEM OF CAYLEY GRAPHS

Pablo Spiga

The isomorphism problem for Cayley graphs has been extensively investigated over the past 30 years. Recently, substantial progress has been made and many new research problems have arisen. The methods used in this area range from group theory through to combinatorial techniques. This talk is devoted to present some results on the classification of Cayley isomorphic groups.

THE LIE STRUCTURE OF A MODULAR GROUP ALGEBRA AND ITS UNIT GROUP

Ernesto Spinelli

Well-known facts show some connections between the Lie structure of a modular group algebra KG and its unit group. The aim of the present talk is to discuss some recent results and open questions in the case in which KG is either Lie nilpotent or Lie solvable.

ON FINITE p-GROUPS ALL NORMAL SUBGROUPS OF WHICH ARE CHARACTERISTIC AND OUTER p-AUTOMORPHISMS AND PERMUTATION REPRESENTATIONS OF FINITE p-GROUPS

Bettina Wilkens

The classification of finite p-groups all normal subgroups of which are characteristic has been an open problem in the theory of infinite p-groups for a while now (see e.g. [2] for an infinite series of examples). A wreath-product construction is sketched by means of which it can be shown that every finite p-group is a subgroup of such a group. It might be of some interest to note that, if $p \neq 2$, the wreath products constructed do possess nontrivial p'-automorphisms. Some results on indecomposable summands of uniserial permutation modules - following the lines laid down in [3] and [4] - emerge on the way.

Addressing the second topic mentioned in the title, we generalise a well-known theorem of Gaschütz's ([1]) stating that every finite p-group has an outer p-automorphism. A theorem is proved giving a description of the finite p-groups P which have a faithful permutation representation of degree $p^n < |P|$ such that $N_{\Sigma_{p^n}}(P)$ induces outer p-automorphisms on P.

Merging the two thematic strands, we conclude with some remarks concerning outer p-automorphisms preserving each conjugacy class of the finite p-group P and make some progress towards classifying those P for which every p-automorphism has this property.

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