Solving Truckload Procurement Auctions Over an Exponential Number of Bundles

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Abstract

Truckload carriers provide hundreds of $billions worth of services to shippers in the U.S. alone each year. Internet auctions provide these shippers with a fast, easy way to negotiate potential contracts with a large number of carriers. Combinatorial auctions have the added benefit of allowing multiple loads to be considered simultaneously in a single auction. This is important because it enables carriers to connect multiple loads in continuous moves or tours, decreasing the empty mileage that must be driven and therefore reducing cost. On the other hand, combinatorial auctions require bidding on an exponential number of loads in order to achieve full economies of scope and scale, which is not tractable except for very small auctions. In most real-world auctions, bidding is limited typically to a very small subset of the potential bids. We present a new approach to combinatorial auctions for truckload procurement that enables the complete set of all possible bids to be considered implicitly, without placing the corresponding burden of an exponential number of bids on either the bidders or the auctioneer. We present the models needed to solve this problem. We then provide extensive computational results to demonstrate the tractability of our approach. Finally, we conclude with numerical analysis to assess the quality of the solutions that are generated.

1 Introduction and Motivation

U.S. freight transportation expenditures in 2005 exceeded $700 billion. Of this amount, $300 billion was accounted for by the truckload segment (American Trucking Association 2007). In many corporations, transportation expenditures can be as high as 30% of the overall cost of goods sold (Ballou 1992); furthermore, trucking is often the dominant cost. Therefore, reducing trucking expenditures can greatly reduce a shipper’s cost of goods sold and improve profitability.

Typically, shippers estimate their freight to be shipped in an upcoming year based on the prior year’s shipments (Foster and Strasser 1991). When contracting out truckload services, a shipper puts forth a request for quotes (RFQ) for a network of loads. Traditionally, carriers (i.e. trucking companies) submitted quotes for individual loads in the RFQ. This is akin to a single-item reverse auction, where each load is awarded independently to a single carrier using a single criterion, usually price (Sheffi 2004).

Today internet auctions provide shippers with a fast and easy way to simultaneously negotiate multiple potential contracts with a large number of carriers. The use of the internet as an auction medium has the benefit of decreasing information-gathering, participation, and transaction costs, as well as increasing geographic and temporal conveniences (Lucking-Reiley 2000). Large corporations such as The Home Depot, Walmart Stores and Staples Inc. rely on applications from software providers such as i2 Technologies, CAPS Logistics, and Logistics.com to procure $billions worth of services annually via internet auctions (Elmaghraby and Keskinocak 2002, Ledyard et al. 2002, and Caplice and Sheffi 2003).

Over the past decade, many of these auctions allow bidders to bid on combinations of loads instead of bidding only on individual loads. Such auctions, called combinatorial auctions, have three stages. First, the auctioneer (on behalf of the buyer) announces multiple loads for bid (henceforth, bid loads) in the auction. Second, the bidders (here, the carriers) submit bids for sets of bid loads (bundles), rather than bidding on each bid load individually. Third, the auctioneer determines the best set of bundles that collectively contain each bid load exactly once, and awards contracts for these bundles (rather than awarding individual bid loads) to the corresponding bidders. In general combinatorial auctions capture the benefits of substitution effects and complementarities, in which the value of a set is not simply the sum of its parts. Using a combinatorial auction in such cases allows bidders to express their true preferences, with the goal of finding better (eg. lower total cost) allocations. This is the case in truckload shipping, due, for example, to the fact that carriers must not only transport the bid loads that they have been awarded, but must also return drivers home. If carriers can string together multiple loads to form a continuous move (tour), then they can decrease their empty mileage and thereby reduce cost.

We illustrate this in Figure 1 with a simple example. Here, we see that the cost of transporting a load from A to B is $lx (the loaded cost per mile times the distance to move the load from A to B) plus $ex (the empty cost per mile times the distance to return the driver home from B to A). The cost of transporting a load from B to A is computed similarly. The cost of transporting both loads, however, is not 2($lx + $ex) but instead only 2$lx, because the two loads can be combined to form a single tour, without any empty mileage. More generally, carriers can improve cost efficiencies by combining loads that are complementary to reduce empty mileage.

Combinatorial auctions allow carriers to bid on bundles of loads that produce these efficient movements. The efficiency gained in combinatorial bidding, in turn, allows carriers to submit more aggressive bids, thereby reducing transportation costs for the shipper.

The benefits of combinatorial auctions have been leveraged in other domains as well. For example, Epstein et al. (2002) applied combinatorial auctions in assigning catering contracts for school meals. Rassenti et al. (1982) and
Figure 1: The network shown in this figure includes one load from A to B and one load from B to A. If x is the distance from A to B and l and e are the loaded and empty costs per mile, respectively, then the total cost of servicing the two individual loads separately is $2lx + 2ex$, while the cost of servicing both loads in a single tour is only $2lx$.


However, two major hurdles remain that prevent the full realization of the benefits of combinatorial auctions. The first is *bid expression and communication*: to fully express economies of scale and scope among all items being auctioned, bidders must construct and submit bids for an exponential number of subsets of these items ($2^n - 1$ for an n-item auction). This is clearly intractable for all but the smallest instances. The second hurdle is in solving the *winner determination problem* (WDP), typically formulated as a *set partitioning problem* (Balas and Padberg 1976), to select the least-cost set of bundles such that each item is in exactly one bundle. WDP is an integer program with an exponentially-large number of binary variables and thus also intractable for all but the smallest instances.

De Vries and Vohra (2003) presented the state of knowledge for solving combinatorial auctions and suggested the use of an “oracle” to alleviate the burden of expressing and communicating an exponential number of bids. The auctioneer invokes the appropriate oracle at any stage of an auction to determine the bid for a particular bundle. Alternatively, an auctioneer may specify a bidding language (Rothkopf et al. 1998, Nisan 2000, Sandholm 2002, Abrache et al. 2002, de Vries and Vohra 2003, Pekec and Rothkopf 2003, and Günlük et al. 2005) to be used by all bidders. Bidding languages may specify ways in which bids must be restricted to a subset of the potential bundles. Alternatively, these bidding languages may allow for full expression of preferences, provided that the preferences have some special structure.

Next, assuming one can overcome the difficulties of bid expression and communication, the auctioneer is still left with solving an exponentially-large WDP to allocate items in the auction and award bundles so that the total cost is minimized. Rothkopf et al. (1998) and De Vries and Vohra (2003) showed that WDP is computationally manageable if the structure of allowable bids permits decomposition into disjoint groups, yields a tractable number of combinations, or results in constraint matrices with integral extreme points. Another strategy is to shift the computational burden of solving WDP to the bidders. Banks et al. (1989), Bykowsky et al. (2000), and Kelly and Steinberg (2000) proposed mechanisms that allow bidders to iteratively submit improvement bids. Finally, Rothkopf et al. (1998), Fujishima et al. (1999), Zurel (2001), Sandholm et al. (2002, 2005), and Günlük et al. (2005) presented provably-fast algorithms and heuristics to efficiently solve WDP under certain conditions. Readers interested in a more comprehensive examination of the theory and applications of combinatorial auctions should refer to a recent book edited by Cramton et al. (2006).

In practice, the hurdles of exponential bidding and an exponentially-large WDP are sometimes circumvented by iterative bidding, restricting bidding to a small number of bundles, and by using exact and approximation algorithms, as discussed in the references above. However, bidding on only a small subset of loads prevents the full realization of the benefits of a combinatorial auction, and incentive compatibility and individual rationality of the auction might be compromised if the auction is not solved to optimality (Ronen 2003).

No single generalized approach can find optimal solutions to fully-enumerated combinatorial auctions for all classes of problems. Nisan (2006) discussed why, in the worst case, a general problem may require exponential communication. Therefore, as highlighted in the preceding paragraph, most research focuses on exploiting problem structure to find acceptable solutions for specific types of auctions. The goal of our research is to show that the underlying structure of a truckload procurement problem can be exploited, enabling us to find solutions to fully-enumerated auctions in practical time frames.

This research extends the existing literature on combinatorial truckload procurement auctions (CTPA). For

In practice, the full benefits of combinatorial auctions for truckload procurement have not yet been achieved. One recent study (Plummer, 2002) showed that only 28 percent of carriers submit bids of more than one load in combinatorial auctions and the majority of these carriers only submit 2-7 multi-load bundles due to practical constraints on bid preparation time, computational resources and technical expertise at their disposal.

Our proposed approach to truckload procurement auctions can (implicitly) capture the full, exponential set of bundles. This approach, which builds on the work of Beil, Cohn and Sinha (2007), leverages the fact that there is a known and amenable structure underlying the cost of servicing a given set of bid loads. Specifically, the least-cost tour (or set of tours) needed to cover a set of loads can be computed by solving a minimum-cost flow problem. We therefore propose to embed this underlying cost structure (which we refer to as a bid generating function) directly into WDP. This eliminates the need for the bidder to compute and communicate an exponential number of bids. Furthermore, we will show that the resulting WDP can be re-formulated as a multi-commodity flow (MCF) problem of polynomial size. Our computational results demonstrate the practical performance of our approach.

Although the primary focus of this approach is on leveraging the complementarities among bid loads and carriers’ existing networks (the salient cost-drivers for carriers), we also expand the formulation to incorporate key operational and practical considerations. We present further computational results to show that the resulting model remains tractable.

The contributions of this research are in:

- developing tractable models to solve a basic truckload procurement auction to optimality in single round, fully considering (implicitly) the exhaustive set of all possible bids;
- showing how the power of mathematical programming can enable this basic problem to be extended to include additional important real-world operational considerations; and
- taking advantage of this new capability to solve fully-enumerated truckload procurement auctions as a tool for conducting numerical analysis on the characteristics of CTPA solutions.

The paper is organized as follows. In Section 2, we formally present the combinatorial auction for truckload procurement. In Section 3, we present basic and enhanced models for this problem, with associated results to demonstrate tractability. In Section 4, we present an alternative model that enables us to significantly increase, under certain conditions, the size of auctions that can be solved while simultaneously improving solution times. In Section 5, we continue to present our computational experiments, focusing on solution characteristics under a variety of conditions. We conclude in Section 6 with a summary of our contributions and our suggestions for future research.

## 2 Combinatorial Auctions for Truckload Procurement

In a basic truckload procurement auction, the auctioneer specifies a set of bid loads, each defined by an origin and destination. Carriers then compute and submit bids for bundles of these loads. Finally, the auctioneer solves a winner determination problem to select bundles and allocate the corresponding loads to winning carriers.

### 2.1 Computing Bundle Bids

In order to understand how carriers compute their bids, we must first understand their cost structure. The obvious cost is the direct movement cost, associated with actually moving a bid load from its origin to its destination. This cost is well understood by the carrier and is largely a function of distance (fuel, equipment depreciation, driver’s wage, tolls, et cetera).

In addition, there are indirect movement costs associated with repositioning a truck from the destination of one load to the origin of the next, so as to form tours. To minimize these costs, and thus improve profitability, carriers must try to build efficient continuous moves with minimal empty mileage. This can be accomplished not only by combining bid loads, but also by taking advantage of a carrier’s pre-existing contracted loads and opportunities on the spot market. For example, Figure 2 shows a sequence of movements for effectively transporting three bid loads.
Backhaul opportunities such as these are usually not known with certainty at the time of the auction. Instead, carriers estimate these opportunities for each directed city pair \((i, j)\) in the network. One way to represent this is with an \(n\)-tiered step function, where each tier corresponds to a different type of backhaul opportunity. For example, one tier might represent the expected number of pre-existing contracted loads (with other shippers) between \(i\) and \(j\), which can be used “for free”, as these movements represent hired rather than empty loads. Another tier might represent the potential for partial connections: pre-existing loads that require the driver to travel empty from \(i\) to some nearby location before picking up the load and delivering it to some location near \(j\), thereby incurring limited empty mileage costs. Estimates of spot market opportunities would be represented by additional tiers as well. Figure 3 provides an example of such a step function. More generally, high-traffic city pairs would have high-capacity, low-cost tiers because of the abundance of backhaul opportunities, while low-traffic city pairs would have lower-capacity, higher-cost tiers representing the decreased likelihood of finding complementary loads.

![Figure 2: A cost-effective tour covering bid loads 1, 2, and 3 that leverages a carrier’s backhaul opportunities.](image)

![Figure 3: Estimate of carrier \(k\)'s backhaul capacity and cost from city \(i\) to city \(j\) using a 4 tiered step-function.](image)

Given a set of bid loads and an estimate of backhaul opportunities, carriers determine the least-cost set of tours to serve these loads, then use this cost in computing their bid price. For example, in a first-price auction, carriers typically bid true-cost plus a percentage-based markup (Song and Regan 2004). Throughout the manuscript, we will assume a first-price auction for the sake of exposition. However, our approach is applicable to other auction mechanisms as well.

### 2.2 Traditional Winner Determination Problem

Once the bids have been submitted, the auctioneer then solves the winner determination problem to select bundles and allocate loads to winning carriers. The traditional winner determination formulation (T-WDP) is as follows:

**Sets**
- \(\mathcal{K}\) the set of carriers
- \(\mathcal{B}\) the set of bid loads
- \(\mathcal{S}^k\) the set of bundles submitted by carrier \(k\), \(\forall k \in \mathcal{K}\)

**Parameter**
- \(p^k_s\) carrier \(k\)'s bid for bundle \(s\), \(\forall k \in \mathcal{K}, s \in \mathcal{S}^k\)
Variable

\( x^k_s \) binary variable that takes value 1 if carrier \( k \) is awarded bundle \( s \) and 0 otherwise, \( \forall \ k \in K, s \in S^k \)

(T–WDP) \[
\min \sum_{k \in K} \sum_{s \in S^k} p^k_s x^k_s \\
\text{subject to:} \sum_{k \in K} \sum_{s \in S^k : l \in s} x^k_s = 1 \quad \forall \ l \in B \tag{2}
\]

\[
\sum_{s \in S^k} x^k_s \leq 1 \quad \forall \ k \in K \tag{3}
\]

\[
x^k_s \in \{0, 1\} \quad \forall \ k \in K, s \in S^k \tag{4}
\]

The objective (1) is to minimize the total cost of procuring truckload services for loads in \( B \). Constraint set (2) states that bundles must be awarded such that each load in \( B \) is in exactly one bundle that is chosen. In a fully-enumerated CTPA, an additional constraint set (3) stating that each carrier can be awarded at most one bundle is imposed (note that each bundle might contain more than one tour). This constraint set is needed to ensure that we do not select a combination of bundles for a given carrier such that, in total, the combination of bundles violates some of the carrier’s operational constraints (for example, using more backhaul capacity than there exists).

We conclude this section by re-iterating the fact that, for practically-sized truckload procurement auctions with thousands of loads, it is of course not possible to explicitly enumerate all \( 2^{|B|} - 1 \) bundle bids. Instead, carriers submit bids for only a small subset of the bundles, due to practical constraints on bid-preparation time, computational resources, and technical expertise available at their disposal (Plummer 2002). As a result, solution quality of CTPAs is often compromised in practice. Furthermore, in a CTPA in which carriers predominantly bid only on bundles with small numbers of loads, we no longer want to impose constraint (3), because it limits our ability to award multiple bid loads to the same carrier (which is not the case when all bundles are enumerated). As a result, it becomes necessary to impose operational constraints explicitly in WDP rather than in the bids. For example, we might need to impose a constraint on each tier of a directed city pair and each carrier, so as to avoid violating capacity constraints.

3 Implicit Truckload Combinatorial Auctions

In the majority of the CTPA literature, bundle prices are assumed to be exogenously endowed. That is, a carrier’s price for a given bundle is assumed to come from some external source. As such, each carrier must explicitly price each bundle of interest individually. In this section, we address the question of how carriers compute bundle prices. We show that these prices can in fact be computed using a well-structured bid-generating function (BGF). Furthermore, the structure of this function can be exploited using an implicit bidding approach (Beil, Cohn, Sinha 2007) to solve WDP using BGF directly, in lieu of the actual bids. This enables the exhaustive set of bundles to be considered implicitly without sacrificing tractability.

3.1 The Bid-Generating Function

Given a bundle of bid loads \( s \), the cost to carrier \( k \) of servicing these loads (and thus the corresponding bid price for this bundle) is comprised of both direct and indirect movement costs. For a given set of loads, the direct movement cost (i.e. cost of moving these bid loads from their origins to their destinations) is more or less fixed and known in advance. The indirect movement cost, on the other hand, depends on the continuous moves that the carrier constructs to minimize empty mileage.

The problem of determining the least-cost set of continuous moves to serve a set of bid loads can be computed by solving a network flow problem. In this formulation, each bid load in the bundle can be viewed as a node. Likewise, we create nodes to represent carrier loads – these represent backhaul opportunities such as pre-existing contracted loads and anticipated opportunities on the spot market. In particular, we construct one node for each unit of capacity in each tier of the step functions corresponding to estimated backhaul opportunities (as described in Section 2.1). Finally, we create arcs representing empty movements between pairs of loads.
The problem is then to create the least-cost set of tours in this network such that each bid load appears exactly once and each carrier load appears at most once. The notation and formulation for this BGF (which we denote by \( f^k \)) are as follows.

**Sets**

- \( B \) exhaustive set of bid loads
- \( C^k \) set of carrier loads (recall that we are currently considering a specific carrier \( k \))
- \( \mathcal{L}^k \) union of sets \( B \) and \( C^k \)
- \( \mathcal{E}^k \) set of arcs \((i, j)\) connecting the destination of load \( i \) to the origin of load \( j \), \( \forall i, j \in \mathcal{L}^k \) (note, \( \mathcal{E}^k \) may include zero-length arcs corresponding to pairs of loads for which the destination of one load is the origin of the other)

**Parameters** (Note that in a first price auction the costs may include a profit markup.)

- \( d^k_l \) direct movement cost of bid load \( l \), \( \forall l \in B \)
- \( c^k_l \) cost to use carrier load \( l \) as part of a continuous move, \( \forall l \in C^k \)
- \( e^k_{ij} \) unit movement cost on arc \((i, j)\), \( \forall (i, j) \in \mathcal{E}^k \)

**Variables**

- \( y^k_{ij} \) binary variable that takes value 1 if carrier \( k \) moves empty on arc \((i, j)\) and 0 otherwise,
  \( \forall (i, j) \in \mathcal{E}^k \)
- \( z^k_l \) binary variable that takes value 1 if carrier load \( l \) is used by carrier \( k \) to complete a continuous move and 0 otherwise, \( \forall l \in C^k \)

Given this notation, we now state BGF –

\[
 f^k(x^s) := \sum_{l \in B} d^k_l x^s_l + \min \left( \sum_{l \in C^k} c^k_l z^k_l + \sum_{(i, j) \in \mathcal{E}^k} e^k_{ij} y^k_{ij} \right)
\]  

subject to:

\[
\sum_{j : (i, j) \in \mathcal{E}^k} y^k_{ij} = x^s_i \quad \forall l \in B \tag{6}
\]

\[
\sum_{i : (i, j) \in \mathcal{E}^k} y^k_{ij} = x^s_l \quad \forall l \in B \tag{7}
\]

\[
\sum_{j : (i, j) \in \mathcal{E}^k} y^k_{ij} = z^k_l \quad \forall l \in C^k \tag{8}
\]

\[
\sum_{i : (i, j) \in \mathcal{E}^k} y^k_{ij} = z^k_l \quad \forall l \in C^k \tag{9}
\]

\[
y^k_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{E}^k \tag{10}
\]

\[
z^k_l \in \{0, 1\} \quad \forall l \in C^k \tag{11}
\]

The objective function (5) states that carrier \( k \)'s price for bundle \( s \) is the sum of the direct movement cost (which depends solely on \( s \)) and the indirect movement cost (which depends on the chosen routing). Constraint sets (6) and (7) state that if bid load \( l \) is in bundle \( s \), then \( l \) must connect from a preceding load and to a following load. Similarly, constraint sets (8) and (9) stipulate that each carrier load chosen to be used as part of a continuous move must connect to and from other loads.

\( f^k(x^s) \) has two important structural characteristics that have significant impact on tractability. These characteristics are described in the following propositions.

**Proposition 1:** The binary restrictions (10) and (11) can be replaced by non-negativity constraints \( y^k_{ij} \geq 0, \forall i, j \in \mathcal{L}^k \) and \( z^k_l \geq 0, \forall l \in C^k \).

**Proof:** The coefficient matrix \( A \) of this problem is totally unimodular, because (1) each element of \( A \) has value 0, 1, or -1, (2) each column contains exactly two non-zero entries and (3) there exists a partition \((M_1, M_2)\), corresponding to constraint sets (6)&(8) and (7)&(9) respectively, of the rows of \( A \) such that each column \( j \) satisfies \( \sum_{i \in M_1} a_{ij} = \sum_{i \in M_2} a_{ij} = 0 \).
\[ \sum_{i \in M_2} a_{ij} \]. In addition, the right-hand-side vector is integer. Thus, the extreme points of this polyhedron are integral (Wolsey 1998) and the integrality of \[ y_{ij}^k \ \forall \ i, j \in L^k \] and \[ z_l^k \ \forall \ l \in C^k \] can be relaxed. \( \square \)

**Proposition 2:** BGF can be reformulated as a circulation problem.

**Proof:** The proof is by construction. Let \( \mathcal{N} \) be the set of all cities, where a city is an origin and/or destination of a bid load in \( \mathcal{B} \). For each load \( l \in s \) with origin \( i \) and destination \( j \), define an arc \( (i, j) \) with per unit cost \( c_{ij} \) (where \( c_{ij} := d_{ij}^k \)) and upper and lower bounds of \( u_{ij} = l_{ij} = 1 \); let \( \mathcal{A}^s \) represent this set of arcs. For each load \( l \in C^k \) with origin \( i \) and destination \( j \), define an arc \( (i, j) \) with per unit cost \( c_{ij} \) (where \( c_{ij} := e_{ij}^k \)), lower bound \( l_{ij} = 0 \) and upper bound \( u_{ij} = 1 \); let \( \mathcal{A}^c \) represent this set of arcs. For each arc \( (i, j) \in E^k \), define an arc \( (i, j) \) with per unit cost \( c_{ij} \) (where \( c_{ij} := e_{ij}^k \)), lower bound \( l_{ij} = 0 \) and upper bound \( u_{ij} = 1 \); let \( \mathcal{A}^l \) represent this set of arcs. Finally, let \( \mathcal{A} := \mathcal{A}^s \cup \mathcal{A}^c \cup \mathcal{A}^l \). Letting \( x_{ij} \) represent the amount of flow on arc \( (i, j) \) for \( (i, j) \in \mathcal{A} \), \( f_k(x^s) \) can be written as:

\[
\min \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij} \\
\text{subject to:} \quad \sum_{j:(i,j) \in \mathcal{A}} x_{ij} - \sum_{j:(j,i) \in \mathcal{A}} x_{ji} = 0 \quad \forall \ i \in \mathcal{N} \tag{12}
\]

\[
\forall (i, j) \in \mathcal{A} \quad l_{ij} \leq x_{ij} \leq u_{ij} \tag{13}
\]

which is a circulation problem. \( \square \)

Circulation problems, which are special cases of minimum cost flow problems, are well known to be easy to solve. For examples of polynomial time algorithms for the circulation problem please refer to Ahuja et al. (1993).

### 3.2 The Implicit Winner Determination Problem

Using the traditional auction mechanism described in Section 2, each carrier must solve a BGF to obtain a bid price for each bundle. For real-world truckload procurement auctions with thousands of loads, constructing bid prices for the full exponential set of bundles is not possible. Furthermore, even if the carriers could compute and communicate bids for all \( 2^{|B|} - 1 \) bundles, the auctioneer could not solve the corresponding exponentially-large WDP. We show that these hurdles can be overcome by using an implicit bidding approach, which directly embeds a carrier’s BGF into WDP. The resulting polynomially-sized model is solution-equivalent to the fully-enumerated T–WDP but, in contrast, is tractable for practically-sized instances.

The thrust of this implicit bidding approach is the following. Rather than submit an exponential number of bundle-price pairs, each carrier \( k \) instead submits its BGF, \( f_k \). The auctioneer can then imbed \( f_k \) directly into WDP, which can be reformulated as:

\[
\min \sum_{k \in \mathcal{K}} f_k(x^k) \\
\text{subject to:} \quad \sum_{k \in \mathcal{K}} x^k_l = 1 \quad \forall \ l \in \mathcal{B} \tag{15}
\]

\[
x^k_l \in \{0, 1\} \quad \forall \ l \in \mathcal{B}, \ k \in \mathcal{K} \tag{16}
\]

Observe that equations (15)-(17) describe a fully enumerated truckload procurement auction where each winner is awarded exactly one bundle (possibly empty). This bundle is described by the single vector of decision variables \( x^k \) (with one element per bid load), taking the place of the set of variables \( x_l^k \) (with one variable per bundle) previously defined in Section 2.2. For each \( l \in \mathcal{B} \), \( x^k_l \) is a binary decision variable that takes value 1 if load \( l \) is assigned to carrier \( k \) and 0 otherwise. The model described by (15)-(17) is solution equivalent to a fully-enumerated T–WDP, as we will show by Proposition 3. Note, however, that in place of \( |\mathcal{K|} (2^{|B|} - 1) \) variables, there are only \( |\mathcal{K|} |\mathcal{B}\) variables. In an auction with 50 cities, 50 carriers, and 200 bid loads, this translates to a reduction from over \( 8 \times 10^{61} \) variables to only 10,000 variables.

Of course, even with this reduction in size, the new formulation may still be quite difficult to solve, depending on the structure of \( f_k \). As we have noted in Section 3.1, however, \( f_k \) is simply a circulation problem. Thus, embedding constraints (5)-(11) in place of the function \( f_k \) leaves us with the following mixed integer program, which we denote
by Implicit Winner Determination Problem 1 (I–WDP1).

\[
\begin{align*}
(I–WDP1) \quad \min & \sum_{k \in K} \sum_{l \in B} d_{kl}^k x_{kl}^k + \sum_{k \in K} \sum_{i \in C^k} c_i^k z_{ik}^k + \sum_{k \in K} \sum_{(i,j) \in E^k} e_{ij}^k y_{ij}^k \\
\text{subject to:} & \quad \sum_{k \in K} x_{kl}^k = 1 \quad \forall \ l \in B \\
& \quad \sum_{j : (i,j) \in E^k} y_{ij}^k = x_{kl}^k \quad \forall \ k \in K, \ l \in B \\
& \quad \sum_{i : (i,j) \in E^k} y_{ij}^k = x_{kl}^k \quad \forall \ k \in K, \ l \in B \\
& \quad \sum_{j : (i,j) \in E^k} y_{ij}^k = z_{ik}^k \quad \forall \ k \in K, \ l \in C^k \\
& \quad \sum_{i : (i,j) \in E^k} y_{ij}^k = z_{ik}^k \quad \forall \ k \in K, \ l \in C^k \\
& \quad x_{kl}^k \in \{0, 1\} \quad \forall \ k \in K, \ l \in B \\
& \quad y_{ij}^k \geq 0 \quad \forall \ k \in K, \ (i,j) \in E^k \\
& \quad z_{ik}^k \geq 0 \quad \forall \ k \in K, \ l \in C^k \\
\end{align*}
\]

(18)

Note that \(x^k\) is no longer a fixed parameter in I–WDP1 but now a vector of decision variables, with constraint (19) assigning each bid load to exactly one carrier (and so, implicitly, defining that carrier’s awarded bundle). Thus, this model assigns bid loads to carriers so as to minimize the resulting least-cost set of continuous moves.

In Proposition 4, we show that this formulation is a special case of the multi-commodity flow problem. Although theoretically difficult (Even et al. 1976), MCF is known to be easy to solve in practice for many real-world instances (Ahuja et al. 1993). This is the case for the truckload procurement auctions, as we will demonstrate through computational results in Section 3.4 and 4.1.

**Proposition 3:** Consider an auction with \(K\) carriers bidding for a set of bid loads \(B\). For each carrier \(k \in K\), if the cost to service a specific bundle \(s\) (where \(s \subseteq B\)) is given by the solution to \(f^k\) (defined by (5)-(11)), then I–WDP1 is solution equivalent to a fully-enumerated T–WDP.

**Proof:** We will show that an optimal solution to T–WDP is a feasible solution to I–WDP1, with equivalent cost, and vice versa. Let \(S^*\) be the set of bundles that correspond to an optimal solution for T–WDP and let \(z_{T–WDP}(S^*)\) be the total cost. Let \((x^*, y^*, z^*)\) be the set of vectors that correspond to an optimal solution for I–WDP1 and let \(z_{I–WDP1}(x^*, y^*, z^*)\) be the total cost.

**Claim 1:** \(z_{T–WDP}(S^*) \geq z_{I–WDP1}(x^*, y^*, z^*)\)

**Proof:** The set of bundles in \(S^*\) satisfies constraint set (19) of I–WDP1. For each \(s \in S^*\), define \(x^s\) to be a vector of size \(|B|\), where \(x_{kl}^s = 1\) if \(l \in s\) and 0 otherwise. Since \(p_s^k\) (price of bundle \(s \in S^*\)) is obtained by solving \(f^k(x^s)\), there exist vectors \((y^s, z^s)\) corresponding to the minimum cost set of carrier loads and empty moves used to complete continuous moves with loads in \(s\). Observe that \((x^s, y^s, z^s)\) satisfies constraint sets (20)-(26) of I–WDP1. If we let \((x, y, z)\) be defined as the concatenation of \((x^s, y^s, z^s) \forall s \in S^*\), then \((x, y, z)\) is a feasible solution to I–WDP1. Since the cost coefficients of \(f^k\) and I–WDP1 are identical, \(z_{T–WDP}(S^*) = z_{I–WDP1}(x, y, z)\). Finally, the optimal solution of I–WDP1 can only be better, thus we must have \(z_{T–WDP}(S^*) = z_{I–WDP1}(x, y, z)\). \(\square\)

**Claim 2:** \(z_{T–WDP}(S^*) \leq z_{I–WDP1}(x^*, y^*, z^*)\)

**Proof:** Constraint set (19) of I–WDP1 and constraint set (2) of T–WDP are equivalent. An optimal solution of I–WDP1, \((x^*, y^*, z^*)\), can be decomposed into \((x^s, y^s, z^s) \forall s \in S^*\). If \(c^*(x^s, y^s, z^s)\) is the total cost of shipping loads in \(s\) using indirect movements corresponding to \((y^s, z^s)\), then \(z_{I–WDP1}(x^s, y^s, z^s) = \sum_{s \in S^*} c^*(x^s, y^s, z^s)\). Observe that indirect movements \((y^s, z^s)\) satisfy constraint sets (6)-(11), and thus \((x^s, y^s, z^s)\) is a feasible solution of \(f^k\). Since the optimal solution of \(f^k(x^s)\) can only do better, we must have \(f^k(x^s) \leq \sum_{s \in S^*} c^*(x^s, y^s, z^s) \forall s \in S^*\). This implies \(\sum_{s \in S^*} f^k(x^s) \leq \sum_{s \in S^*} c^*(x^s, y^s, z^s) = z_{I–WDP1}(x^*, y^*, z^*)\). Since \(z_{T–WDP}(S^*) = \sum_{s \in S^*} f^k(x^s)\), we have \(z_{T–WDP}(S^*) \leq z_{I–WDP1}(x^*, y^*, z^*)\). \(\square\)

**Proposition 4:** I–WDP1 can be reformulated as a multi-commodity flow problem.
PROOF: The proof is by construction. Let \( \mathcal{N} \) be the set of all cities, where a city is an origin and/or destination of a bid load in \( \mathcal{B} \). For each carrier \( k \in \mathcal{K} \) and bid load \( l \in \mathcal{B} \) with origin \( i \) and destination \( j \), define an arc \((i, j)\) with per unit cost \( c_{ij}^k \) (where \( c_{ij}^k := d_{ij}^k \)), lower bound of \( l_{ij}^k = 0 \), and upper bound \( u_{ij}^k = 1 \); let \( A^B \) represent this set of arcs. For each carrier \( k \in \mathcal{K} \) and carrier load \( l \in \mathcal{C}^k \) with origin \( i \) and destination \( j \), define an arc \((i, j)\) with per unit cost \( c_{ij}^k \) (where \( c_{ij}^k := e_{ij}^k \)), lower bound of \( l_{ij}^k = 0 \), and upper bound \( u_{ij}^k = 1 \); let \( A^{(k)} \) represent this set of arcs. For each carrier \( k \in \mathcal{K} \) and empty movement arc \((i, j) \in \mathcal{E}^k \), define an arc \((i, j)\) with per unit cost \( c_{ij}^k \) (where \( c_{ij}^k := e_{ij}^k \)), lower bound of \( l_{ij}^k = 0 \), and upper bound \( u_{ij}^k = 1 \); let \( A^{(k)} \) represent this set of arcs. Finally, for each carrier \( k \in \mathcal{K} \), let \( A^k := A^{B(k)} \cup A^{(k)} \cup A^{(k)} \). Letting \( x_{ij}^k \) represent the amount of flow of commodity (i.e. carrier) \( k \) on arc \((i, j)\) for \( k \in \mathcal{K} \) and \((i, j) \in A^k \), I–WDP1 can be written as:

\[
\min \sum_{k \in \mathcal{K}} \sum_{(i, j) \in A^k} c_{ij}^k x_{ij}^k 
\]

subject to: \( 1 \leq \sum_{k \in \mathcal{K}} x_{ij}^k \leq 1 \) \( \forall (i, j) \in A^B \) \( (27) \)

\[
\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{K}} x_{ij}^k - \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{K}} x_{ij}^k = 0 \quad \forall k \in \mathcal{K}, i \in \mathcal{N} \) \( (28) \)

\[
l_{ij}^k \leq x_{ij}^k \leq u_{ij}^k \quad \forall k \in \mathcal{K}, (i, j) \in A^k \) \( (29) \)

\[
l_{ij}^k \leq x_{ij}^k \leq u_{ij}^k \quad \forall k \in \mathcal{K}, (i, j) \in A^k \) \( (30) \)

(27)-(30) is a multi-commodity flow problem. \( \square \)

Multi-commodity flow problems typically have strong LP relaxations and thus result in a smaller tree for branch-and-bound. Additionally, the embedded network structure of MCF’s can be exploited by solution algorithms, resulting in efficient solution times even for large instances. This is demonstrated by our computational results in subsequent sections, which show tractability for auctions with up to 100 cities, 50 carriers (bidders), and 5000 bid loads.

### 3.3 Privacy Issues of Implicit Bidding Approach

Using the implicit bidding approach, each carrier submits a BGF to the auctioneer, so naturally, carriers may be concerned about providing private information that could be exposed to competitors. Similarly, the auctioneer (shipper) may have concerns about false-name bidding and other forms of collusion.

We believe, given the benefits of achieving full economies of scope in a fully-enumerated CTPA, that carriers and shipper have significant incentives to overcome these concerns. Consider an analogous example, Vendor Managed Inventory (VMI) systems (Chopra and Meindl 2007), in which a retailer provides its suppliers with direct visibility to inventory levels. Like our proposed approach, VMI raises concerns about information privacy and security. However, VMI systems are widely used today by large corporations such as Walmart and The Home Depot because of the substantial benefits they provide (Lee et al. 1997).

In practice, third party service providers such as Manhattan Associates and Ariba can provide services such as bidder pre-qualification and transaction confidentiality to improve information security and privacy and to limit the risk of information leakage. Additionally, emerging research on cryptographically-secured auctions (Kudo 1998, Franklin and Reiter 1996) provides an additional way to protect private information. We believe our proposed method provides significant incentives for its use and as such may galvanize deployment of existing, or development of new, infrastructures.

### 3.4 Computational Experiments and Results

The following computational experiments are developed to assess the viability of our approach for CTPAs. Computational experiments were conducted on a Sun x4600-M2 with 8 AMD Opteron 2818 processors and 64 GB of RAM. The test machine was running Red Hat Enterprise Linux 4. Models and algorithms were coded using C++ and ILOG Concert Technology and solved using ILOG CPLEX 10.0.

We provide results for auctions with 50, 100, and 200 bid loads. The solution times reported for each set of auctions are averages of 100 randomly generated instances. For each instance, the number of carriers is fixed at 50. Loads were generated across a network of 50 cities randomly selected from a list of the 100 most populous cities in the United States. Each carrier’s cost structure was also randomly generated. The direct movement cost is based on a randomly-generated loaded cost-per-mile using a Normal \((1.10, 0.05^2)\) random variable. A carrier’s cost to transport
A bid load is equal to the distance from origin to destination times the carrier’s loaded cost per mile. The indirect movement cost is comprised of empty moves and carrier loads. The cost of an empty move is equal to the distance traveled times the carrier’s empty cost per mile, which is randomly generated using a Normal \((0.80, 0.05^2)\) random variable. The set of carrier loads \(C^k\) consists solely of pre-existing contracted loads at a cost of \(\$0\) per load. The number of carrier loads for a carrier is uniformly generated to be between 5-15 percent of the number of bid loads in the auction. Carrier loads for each carrier are constructed by randomly selecting origin-destination pairs for the network of 50 cities. The distance between two cities is calculated using the Haversine formula (giving great-circle distances between two points on a sphere from their longitudes and latitudes).

Figure 4: I–WDP1 solution times for auctions of size 50, 100, and 200 bid loads.

Figure 4 shows solution times for auctions with 50, 100, and 200 loads. Instances with 50 loads solved in an average time of 8 seconds across the 100 instances. Auctions with 100 loads solved on average in 65 seconds, while auctions with 200 loads solved on average in 819 seconds. Overall, there was very little branching for these three sets of runs, with the average number of branch-and-bound nodes for each of the three sets of instances being 1, 3, and 19, respectively. This is not surprising, given I–WDP1’s multi-commodity flow structure.

These results suggest that we can in fact solve to optimality fully-enumerated CTPAs of up to 200 loads with relative ease. We contrast this with the traditional approach, which would require each carrier to compute and submit more than \(1.1 \times 10^{15}\) bundle bids and the auctioneer to solve a T–WDP with \(5.6 \times 10^{16}\) binary variables for our smallest computational experiment - clearly, an intractable task.

### 3.5 Operational Considerations

In addition to the basic constraints shown in (18)-(26), shippers and carriers may have other operational considerations to take into account. Although all models are, of course, simplifications of the real world, we are able to expand our formulation to capture key operational considerations:

- **Number of loads**: The shipper and carriers may want to specify a minimum and maximum number of loads that each carrier can be awarded. From the shipper’s perspective, a minimum ensures that each winning carrier is awarded enough business to justify the cost of maintaining a shipper-carrier relationship. A maximum ensures that the risk of transporting loads is adequately spread across many carriers. From the carrier’s perspective, a minimum ensures that the carrier wins at least a threshold volume, and a maximum ensures that the carrier’s capacity is not violated.

These constraints can be modeled as follows, where \(l_{\min}^k\) is the minimum number of loads carrier \(k\) must win (or nothing), \(l_{\max}^k\) is the maximum number of loads carrier \(k\) can win, and \(w^k\) is a binary variable that takes value 1 if carrier \(k\) is awarded at least one load and 0 otherwise.

\[
w^k \leq \sum_{l \in B} x_{il}^k \leq |B| w^k \quad \forall k \in K
\]  

(31)
\[ t_{min}^k w^k \leq \sum_{l \in B} x_{l}^k \leq t_{max}^k w^k \quad \forall k \in K \tag{32} \]

- **Number of winners**: The shipper might prefer to award loads to no fewer than \( i_{min} \) carriers and to no more than \( i_{max} \) carriers, thus ensuring a manageable number of vendor relationships and adequate spreading of risk. We can model such restrictions as follows, where \( w^k \) is a binary variable that takes the value 1 if carrier \( k \) is awarded at least one load and 0 otherwise.

\[ w^k \leq \sum_{l \in B} x_{l}^k \leq |B| w^k \quad \forall k \in K \tag{33} \]

\[ i_{min} \leq \sum_{k \in K} w^k \leq i_{max} \tag{34} \]

- **Total mileage**: Another alternative to restricting the number of loads a carrier can be assigned is to restrict the total mileage a carrier can be awarded. The reasons for doing so are similar to those for restricting the number of loads. Such constraints can be modeled as follows, where \( \beta_{min}^k \) is the minimum number of miles carrier \( k \) must cover (or nothing), \( \beta_{max}^k \) is the maximum miles carrier \( k \) can cover, \( m_{ij}^k \) denotes the distance from the origin of load \( l \) to its destination, and \( n_{ij} \) is the distance from the destination of load \( i \) to the origin of load \( j \).

\[ \beta_{min}^k w^k \leq \sum_{l \in B} m_{ij}^k x_{l}^k + \sum_{l \in C_k} m_{i}^k z_{l}^k + \sum_{(i,j) \in \mathcal{E}_k} n_{ij} y_{ij}^k \leq \beta_{max}^k w^k \quad \forall k \in K \tag{35} \]

- **Favoring of incumbents**: There is a cost to the shipper to start a relationship with a new carrier. The shipper can account for this cost by adjusting a carrier’s bid price. In practice, incumbents are often favored by 3-5 percent – especially for service-critical or time-sensitive loads (Caplice and Sheffi 2003). This constraint can be accounted for by simply adjusting a new carrier’s cost coefficient by a constant or multiplicative factor \( \delta^k \).

\[ w^k \leq \sum_{l \in B} x_{l}^k \leq |B| w^k \quad \forall k \in K \tag{36} \]

\[ \delta^k w^k + \left( \sum_{l \in B} d_{ij}^k x_{l}^k + \sum_{l \in C_k} c_{ij}^k z_{l}^k + \sum_{(i,j) \in \mathcal{E}_k} e_{ij}^k y_{ij}^k \right) \tag{37} \]

or

\[ \delta^k \left( \sum_{l \in B} d_{ij}^k x_{l}^k + \sum_{l \in C_k} c_{ij}^k z_{l}^k + \sum_{(i,j) \in \mathcal{E}_k} e_{ij}^k y_{ij}^k \right) \tag{38} \]

- **Performance measures**: The level of service (such as percentage on-time, claims performance, acceptance rate, et cetera) provided by a carrier can also be incorporated in the auction by adjusting the carrier’s cost coefficient by a constant or multiplicative factor (see (36)-(38)).

To consider the impact of these constraints on performance, we again consider auctions with 50, 100, and 200 bid loads, randomly generating 10 instances for each case. Figure 5 presents average solution times for the unconstrained I–WDP1, I–WDP1 with constraints on the number of winners, I–WDP1 with constraints on the number of loads per winner, and I–WDP1 with constraints on both the number of winners and the number of loads per winner. We constrain the number of winners to be at least 10 carriers and at most 15 carriers. We constrain the minimum and maximum number of loads that a carrier can be awarded to be at least 5 percent of the total number of bid loads and at most 40 percent of the total number of bid loads.

Our computational results in Figure 5 show that I–WDP1 remains tractable even with constraints on the number of winning carriers and the number of loads per carrier. As expected, solution time increased with the addition of operational constraints, particularly with constraints on the number of loads per winner. We speculate that this is largely due to the violation of a pure network flow structure. There is a substantial body of literature devoted to solution techniques for network flow problems with side constraints (Gersht and Shulman 1987, Belling-Seib et al. 1998, Holmberg and Yuan 2003), leading us to believe that further improvements in the solution times of I–WDP1 with side constraints are attainable.
4 A Large-Scale Implicit Winner Determination Problem

Computational results for I–WDP1 showed tractability for CTPAs with up to 200 bid loads. However, as the number of bid and carrier loads in the auction increases, the solution time of I–WDP1 grows substantially: in our experiments, doubling the number of loads on average increases solution time by an order of magnitude (Figure 4). This increase in computation time is attributed to the exponential increase in the size of I–WDP1. For the unconstrained I–WDP1, doubling the number of bid and carrier loads doubles the number of binary variables, quadruples the number of continuous variables and doubles the number of flow balance constraints (20-23).

The scalability of I–WDP1 is hindered by the need to balance network flows for each load in the network. In this section, we introduce an alternative implicit WDP model, I–WDP2, that enforces flow balance at the nodes (here, cities) and aggregates loads with common origin and destination cities into a single lane with a corresponding weight representing the number of loads on that lane. We also create arcs that can be thought of as carrier loads. In particular, we construct one arc for each tier of the step function corresponding to estimated backhaul opportunities (as described in Section 2.1). Finally, we create arcs representing empty movements between pairs of nodes. I–WDP2 is largely invariant to increases in the number of loads, thus yielding scalability improvement of orders of magnitude. The formulation is presented below:

Sets

\( \mathcal{N} \) set of nodes (cities) corresponding to arc origins or destinations
\( \mathcal{K} \) set of carriers
\( \mathcal{A}^B \) set of bid load arcs defined for all directed city pairs \((i, j), \forall i, j \in \mathcal{N}\)
\( \mathcal{A}^E \) set of empty movement arcs defined for all directed city pairs \((i, j), \forall i, j \in \mathcal{N}\)
\( \mathcal{A}^C_k \) set of carrier arcs that correspond to backhaul capacities for carrier \(k, \forall k \in \mathcal{K}\)

Parameters

\( O(a) \) origin of arc \(a, \forall a \in \mathcal{A}^B \cup \mathcal{A}^E \cup \mathcal{A}^C_k \)
\( D(a) \) destination of arc \(a, \forall a \in \mathcal{A}^B \cup \mathcal{A}^E \cup \mathcal{A}^C_k \)
\( d^B_{ki} \) direct movement cost of bid load \(l\) for carrier \(k, \forall k \in \mathcal{K}, l \in \mathcal{B}\)
\( c^C_{ka} \) transportation cost of arc \(a\) for carrier \(k, \forall k \in \mathcal{K}, a \in \mathcal{A}^C_k \)
\( c^E_{ia} \) transportation cost of arc \(a\) for carrier \(k, \forall k \in \mathcal{K}, a \in \mathcal{A}^E \)
\( w^t_a \) capacity of arc \(a\) for carrier \(k, \forall k \in \mathcal{K}, a \in \mathcal{A}^C_k \)
\( w_a \) weight (i.e. the number of the number of bid loads) of arc \(a, \forall a \in \mathcal{A}^B \)

Variables

\( x^B_{ka} \) number of loads on arc \(a\) awarded to carrier \(k, \forall k \in \mathcal{K}, a \in \mathcal{A}^B \)
Proposition 5: I–WDP (by Proposition 4).

The number of carrier loads used to complete tours must be less than or equal to the capacity available. Constraint set (44) ensures flow conservation of nodes (here, cities) in the network; that is, the number of indirect movements. Lane cover constraint set (40) stipulates that all loads in a given lane must be covered by carrier(s). The objective function (39) minimizes the total cost attributed to direct and usage of carrier loads, respectively.

Claim 1

We will show that an optimal solution to I–WDP1 is a feasible solution to I–WDP2, with equivalent cost.

Proof: Since, the optimal solution of I–WDP2 can only do better, we have $z^*_{I-WDP1} \leq z^*_{I-WDP2}$. \hfill \Box

Claim 2

Each lane in I–WDP2 can be decomposed into individual bid loads. Since each load is define by an origin and destination, flow balance at the nodes (here, cities) implies flow balance for each load (bid and carrier). A carrier’s backhaul capacity at each tier $u_k^k$ is decomposable into individual carrier loads. The objective coefficients of I–WDP1 and I–WDP2 are the same; therefore, the optimal solution to I–WDP1 is a feasible solution to I–WDP2 with the same cost. Since, the optimal solution of I–WDP2 can only do better, we have $z^*_{I-WDP2} \leq z^*_{I-WDP1}$. \hfill \Box

I–WDP2 has three sets of variables, $x_{a}^k$, $y_{a}^k$ and $z_{a}^k$, representing bid load assignments, empty movements, and usage of carrier loads, respectively. The objective function (39) minimizes the total cost attributed to direct and indirect movements. Lane cover constraint set (40) stipulates that all loads in a given lane must be covered by carrier(s). Constraint set (44) ensures flow conservation of nodes (here, cities) in the network; that is, the number of movements into a node must be equal to the number of movements out of the node. Constraint set (42) states that the number of carrier loads used to complete tours must be less than or equal to the capacity available.

In the following proposition, we show that I–WDP2 is solution-equivalent to I–WDP1 and hence equivalent to T–WDP (by Proposition 4).

Proposition 5: I–WDP1 and I–WDP2 are solution-equivalent; that is, if $z^*_{I-WDP1}$ is the optimal objective value of I–WDP1 and $z^*_{I-WDP2}$ is the optimal objective value of I–WDP2, then $z^*_{I-WDP1} = z^*_{I-WDP2}$.

Proof: We will show that an optimal solution to I–WDP1 is a feasible solution to I–WDP2, with equivalent cost, and vice versa.

Claim 1: $z^*_{I-WDP1} \leq z^*_{I-WDP2}$

The number of bid loads $l \in B$ with a common origin and destination is equal to $w_a$, $\forall a \in A^B$. Flow balance for I–WDP1 is satisfied by each bid load $l \in B$, therefore, flow balance at each node (here, a city) is also satisfied. Since we defined one carrier load for each unit of capacity in each tier of a step function, the sum of all carrier loads representing backhaul capacity in tier $a$ must equal $w_a$, $\forall a \in A^B$, $k \in K$. The objective coefficients of I–WDP1 and I–WDP2 are the same. Therefore, the optimal solution to I–WDP1 is a feasible solution to I–WDP2 with the same cost. Since, the optimal solution of I–WDP2 can only do better, we have $z^*_{I-WDP1} \leq z^*_{I-WDP2}$. \hfill \Box

Claim 2: $z^*_{I-WDP2} \leq z^*_{I-WDP1}$

Each lane in I–WDP2 can be decomposed into individual bid loads. Since each load is define by an origin and destination, flow balance at the nodes (here, cities) implies flow balance for each load (bid and carrier). A carrier’s backhaul capacity at each tier $u_k^k$ is decomposable into individual carrier loads. The objective coefficients of I–WDP1 and I–WDP2 are the same; therefore, the optimal solution to I–WDP1 is a feasible solution to I–WDP2 with the same cost. Since, the optimal solution of I–WDP2 can only do better, we have $z^*_{I-WDP2} \leq z^*_{I-WDP1}$. \hfill \Box

Claim 1 and Claim 2 together imply that $z^*_{I-WDP1} = z^*_{I-WDP2}$. \hfill \Box

Two things contribute to increasing the scalability of I–WDP2. First, variables $x_{a}^k$ ($a \in A^{|B|}$) now represent the assignment of bid loads on a directed city pair as opposed to individual bid load assignments. By aggregating loads with common origin and destination cities into a single variable we reduce the size of I–WDP2 and further, eliminate much of the symmetry that exists in the branch-and-bound tree caused by loads with a common origin and destination. Secondly, I–WDP2 is invariant to increases in the number of bid loads and backhaul capacity. Increasing the number of bid loads simply increases the cover constraint parameter $w_a$ ($a \in A^{|B|}$) of constraint set (40), while increasing backhaul capacity only increases the upper bound of arcs in $A^C_k$ of constraint set (42).
4.1 Computational Experiments and Results

The following computational study is developed with two objectives. The first is to demonstrated the scalability of I–WDP2; the second is to show that I–WDP2 with additional operational constraints remains tractable. Hardware and software setup are as described in Section 3.4. We provide results for auctions with 1000, 2000, and 5000 bid loads and 50 carriers bidding. Unless otherwise stated, the solution characteristics reported for each set of auctions were averages of 100 randomly generated instances. Bid and carrier loads are randomly generated on a network consisting of the 100 most populous cities in the United States.

Figure 6: I–WDP2 solution times for auctions of size 1000, 2000, and 5000 bid loads.

Figure 6 shows solution time characteristics for the three auctions sizes. On average auctions with 1000, 2000, and 5000 bid loads solved in 411, 194, and 69 seconds respectively. The median times reported are substantially lower than the averages which indicates that average solution times are skewed by a few long running instances. It is interesting to note that average solution times are inversely proportional to the size of the auction. All else being equal, increasing the number of loads in the auction actually improves solution time. Intuitively, given a fixed-size network with uniformly distributed loads, increasing the number of loads in the auction improves the probability of finding complementary loads. Therefore, for large auctions, the cost of the auction is primarily dominated by direct movement costs and the majority of bid loads are allocated to a few low-cost carriers.

The solution times suggest that we can in fact solve to optimality fully-enumerated CTPAs of up to 5000 bid loads with relative ease. We contrast this against the traditional approach, which would require each carrier to compute and submit more bundle bids than there are atoms in the universe and the auctioneer to solve a T–WDP with a corresponding number of binary variables - clearly, an intractable task.

To consider the impact of operational constraints on performance, we again consider auctions with 1000, 2000, and 5000 bid loads, randomly generating 10 instances for each case. Figure 7 presents average solution times for the unconstrained I–WDP2, I–WDP2 with constraints on the number of winners, I–WDP2 with constraints on the number of loads per winner, and I–WDP2 with constraints on both the number of winners and the number of loads per winner. We constrain the number of winners to be at least 10 carriers and at most 15 carriers. The minimum and maximum number of loads that each carrier can be awarded is at least 5 percent and at most 40 percent of the total number of bid loads, respectively.

As expected, solution time again increased significantly with the addition of both sets of constraints. However, despite these additional constraints, I–WDP2 remains tractable for auctions with up to 5000 bid loads, with increasing tractability as the number of bid loads in the auction grows. The results represented in Figure 7 are obtain using default CPLEX solver settings and no preprocessing routines. This leads us to believe that further improvements in solution times of I–WDP2 with side constraints are attainable.
5 Numerical Analysis

In the preceding sections, we presented an implicit bidding approach and models for solving fully enumerated CTPAs to optimality and used randomly generated data to assess tractability. We demonstrated the viability of this approach by presenting computational results for CTPAs with up to 5000 bid loads. Furthermore, we showed that these models can be extended to account for practical considerations and still maintain tractability.

Now that we have a tractable way to solve, in a single round, fully enumerated CTPAs to optimality (which was not possible in the past) we can also conduct numerical analysis to better understand the performance and characteristics of practical CTPAs. To our knowledge, in the literature there has not appeared such a study of fully enumerated CTPA outcomes.

In particular, we consider the following questions:

- How does the number of loads affect solution characteristics?
- How do differences in network structure affect solution characteristics?
- What is the relationship between constraints on the number of carriers and loads per carrier versus overall cost?

Effect of Number of Loads on Solution Characteristics

We first consider how varying the number of bid loads and carriers’ backhaul capacities impacts solution time, the number of winners, and the percentage of empty movement, where percentage of empty movement is defined as follows:

\[
\text{percentage of empty movement} = \frac{\text{total empty movement cost}}{\text{total direct and indirect movement cost}} \times 100
\]

In this numerical experiment, we again considered auctions of 1000, 2000 and 5000 bid loads. The number of carriers and nodes (here, cities) is fixed at 50 and 100 respectively. Bid loads and carriers’ backhaul capacities are randomly generated on the network. For each of these auction sizes, we hold the number of bid loads constant and vary the backhaul capacity of each carrier as a percentage of the number of bid loads, ranging from zero to 500 percent. In Figures 8-10, we present results for backhaul capacities ranging from zero to 40, 100, and 35 percent respectively. Beyond these ranges (up to 500 percent) the three graphs remain relatively flat.

Our computational results show that CTPAs have special properties at two extremes: when carriers have very small or very large backhaul capacities. At these extremes, the WDP is extremely tractable as evident by the small solution times in Figure 8. Furthermore, at these extremes the number of winner is relatively small (Figure 9). Intuitively, when carriers have very little backhaul capacity, the cost of the auction is dominated by the carriers’ direct
and empty movement costs, therefore, a small number of low-cost carriers typically win the majority of the bid loads. As carriers’ backhaul capacities increase, the likelihood of finding cost-effective connections also increases, leading to decreases in empty movements (Figure 10). As this happens, the majority of continuous movements are formed by combining bid loads with carrier loads. In this case, the cost of the auctions is dominated by direct movement cost and a small number of carriers with low direct movement costs typically win out.

**Effect of Network Structure on Solution Characteristics**

Next, we evaluate the impact of network structure on solution characteristics. Specifically, what is the impact on solution time, number of winners, and the percentage of empty movement? We consider a network with 100 nodes (here, cities) divided into six regions, where each bidder is either a national or regional carrier. National carriers have backhaul capacities that are uniformly dispersed throughout the entire network, while regional carriers have backhaul capacities that are concentrated in one specific region. Bid loads are generated in either a uniform network, where bid loads have randomly selected origin and destination cities, or in a hub-and-spoke network, where bid loads originate from one of three hubs (selected a priori) and terminate at a random node. An example of a hub-and-spoke network and an uniform network is provided in Figure 11.
The computational results presented below are based on CTPAs with 1000 bid loads and 50 carriers, each with a backhaul capacity of 100 carrier loads. We compare computational results for the following setups: national carriers with uniform bid loads, regional carriers with uniform bid loads, national carriers with hub-and-spoke bid loads and regional carriers with hub-and-spoke bid loads. For each of the four setups, results are averages across ten randomly generated instances.

Figure 12 shows that in a very unstructured network, with national carriers and uniform bid loads, the average solution time is 411 seconds. In contrast, with a very structured network consisting of only regional carriers and hub-and-spoke bid loads the average solution time is only 12 seconds. In the first setup, there is significant fractionality (average number of branch-and-bound nodes is 318); carrier characteristics, in terms of costs and backhaul capacity, are very homogenous and bid loads are uniformly generated throughout the network. In the final setup, there is less fractionality (average number of branch-and-bound nodes is 0) as carriers are more heterogeneous; each regional carrier has backhaul capacity that is concentrated in a specific region of the network. Observe that the computational results presented earlier in Section 3 and Section 4 are based on the least tractable setup, with national carriers and uniform bid loads. As such, we can expect computational performance of our approach to be even better in practical networks with more structure.

Figure 13 shows that CTPAs on networks with hub-and-spoke bid loads incur a higher empty movement cost. This is as expected, since carriers must sometimes return empty to hubs to pick up a bid load. With respect to the number of winners, less structure implies fewer winners and more structure implies more winners. On less structured networks, a few of the lower cost carriers typically dominate, while on a more structured network, the unique set of
backhaul capacities that each carrier brings to the auction plays a key role in forming cost effective movements and so more carriers are likely to be allocated bid loads.

Effects of Imposed Constraints on Overall Cost

In the first numerical experiment, we observed (Figure 9) that for large numbers of carrier loads the number of winners is very small and for small numbers of carrier loads, the number of winners is very large. This is not always desirable from an operational standpoint. In this numerical experiment we assess the relationship between imposed constraints on number of winners and the number of loads per winner versus the overall cost.

We considered three auctions of 1000 bid loads (100 cities, 50 carriers). For each instance, we constrained the minimum number of loads awarded to each winner to be at least ten. Initially, each of these three instances are solved with no constraint on the number of winners. The number of winners in the optimal solutions are then used as baselines for this experiment. For each of these instances, we re-solved the WDP multiple times, imposing different
6 Conclusions and Future Research

In this paper, we applied an implicit bidding approach to solve CTPAs in a single round while implicitly considering the exhaustive set of all possible bundles. This approach directly addresses the two main challenges of combinatorial auctions: bidding on an exponentially-large set of bundles and solving the corresponding exponentially-large WDP. Using the implicit bidding approach, instead of submitting an exponential number of bundles, each carrier simply submits a BGF, which is embedded directly into the WDP. We showed that in truckload transportation, a carrier’s BGF is a minimum-cost flow problem and the resulting WDP is a multi-commodity flow problem, which is generally known to be tractable in practice. Tractability was demonstrated through extensive computational experiments for auctions with up to 5000 bid loads. Furthermore, WDP can be extended to include additional important real-world operational considerations while preserving tractability. In short, we presented a new approach and models for solving CTPAs to optimality that are computationally efficient, consider the exhaustive set bundles, and achieve full economies of scope, which is not possible with current approaches.

Further, we took advantage of this new capability to solve fully-enumerated CTPAs to optimality as a tool for conducting numerical analysis on the quality and characteristics of solutions. We showed that, using this approach, shippers can conduct numerical experiments to assess how CTPA characteristics (e.g., solution time, number of winners, empty movement percentages, et cetera) are likely to change with important problem parameters (e.g., number of carriers, number of loads, carriers’ backhaul capacities, et cetera). Additionally, the shipper can use our approach to perform what-if analysis to assess the cost impact of imposing various operational constraints before finalizing contracting decisions.

In terms of future work, we envision two types of research. First, extensions are possible for our work in CTPAs. For instance, additional operational considerations could be addresses, such as regional coverage requirements, backup carrier bids, and maximum tour length constraints. Maximum tour length constraints are applicable because drivers...
must return home within a limited time window. This constraint set can only be addressed with explicit knowledge
of bid load allocations and the tours constructed to cover these allocated loads. We are currently addressing this
problem using column generation to solve a tour based model, where each variable represents a viable tour or set of
tours.

Furthermore, now that we can solve CTPAs in a reasonable amount of time, we can use this tool to assess
the quality of various auction mechanisms for truckload procurement. Specifically, how would different auction
mechanisms (first price, second price, et cetera) perform under various procurement settings? Even with just a single
item, revenue–or cost–equivalence between standard auction formats fails if bidders are asymmetric.

Additionally, uncertainties in the cost parameters exist due to spot market variability, carriers’ uncertainties about
their existing and future networks, and timing effects; detailed modeling of such uncertainties and development of
appropriate solution approaches are interesting, but challenging, directions for future research.

Secondly, future work could extend the use of the implicit bidding approach to other application domains. Of
particular interest initially is the identification of domains for which the bid generating approach appears amenable.
A sample of potential domains include wireless spectrum auctions, energy auctions, and procurement auctions with
capacity-constrained suppliers.

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