A carrier’s optimal bid generation problem in combinatorial auctions for transportation procurement

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Abstract

We consider the carrier’s optimal bid generation problem in combinatorial auctions for transportation procurement. Bidders (carriers) employ vehicle routing models to identify sets of lanes (origin-destination pairs) based on the actual routes that a fleet of trucks will follow in order to maximize profit. Routes are constructed by optimally trading off repositioning costs of vehicles and the rewards associated with servicing lanes. The carrier optimization represents simultaneous generation and selection of routes and can incorporate any existing commitment. We employ both column generation and Lagrangian based techniques for solving the carrier optimization model and present numerical results.

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Keywords: Transportation procurement; Carrier bid generation; Combinatorial auction; Vehicle routing; Branch and bound; Integer programming decomposition

1. Introduction

Truckload (TL) transportation accounts for a substantial portion of the annual overall billion dollar freight transportation industry in North America. A significant portion of this demand emanates from shippers e.g. manufacturers and retailers that need freight to be transported in dedicated (truckload) movements from origins to destinations. Shippers often need to acquire third party logistics carriers to meet demand since in-house transportation capacity may be insufficient. Usually, contracts for transportation services with carriers are sought through a competitive request for proposals (RFP) process. The basic unit of interest is called a lane which specifies an origin-destination pair for particular freight movement.

Carriers usually engage in some form of auction to bid on individual lanes of interest. Shippers evaluate bids on lanes individually and then awards lanes separately to carriers based on several factors e.g. price and business requirements. However, as noted by Caplice (1996), carriers are often driven by economies in scope in obtaining lanes. That is, a carrier may desire to bid on separate lanes that collectively represent a cost...
efficient service route with respect to minimizing empty miles travel and other repositioning costs. For example, a contract to move freight from Toronto to Montreal would be not be worth as much as a contract that requires movement of freight from Toronto to Montreal and then movement of freight from Montreal and back to Toronto. The first case would involve a trip back to Toronto with an empty load. In general, repositioning empty trucks comprise a substantial portion of operating expenses. In short, carriers seek a set of lanes that are synergistic with respect to repositioning costs. However, many current forms of transportation procurement do not guarantee that a carrier may get a particular cost minimizing set of lanes. A carrier may end up with an incomplete set of lanes that is worth far less than having the complete set or a carrier may incur a loss in servicing the incomplete set of won lanes due to repositioning costs. In auction parlance, the problem of not obtaining a complete set of desired items in a multi-item auction is called the exposure problem (see Bykowsky et al., 1995; Kwasnica et al., 2005). Transportation procurement auctions in most instances award lanes separately and so may be subject to the exposure problem. In general, it is desirable for auction outcomes to result in efficient allocations where bidders that value the items the most obtain them and pay accordingly the highest for the items. The exposure problem can prevent auctions from achieving high levels of efficiency since bidders may be reluctant to bid aggressively for items in fear of not obtaining a complete set of items which can make generation of revenue difficult for the auctioneer and allow others to obtain items for lower prices.

Recently, combinatorial auctions (CA) have been suggested to overcome the exposure problem in auction settings (Kwasnica et al., 2005). Combinatorial auctions are auction formats that allow bidders to place a single bid on a set of distinct items. In the context of transportation procurement carriers could place a single bid on several distinct lanes and if a bid were successful then the carrier would receive the complete set (package) else carriers would receive nothing. This would minimize the risks in obtaining only a subset of lanes that are not worth much. It would also be possible for a carrier to submit multiple packages each consisting of one or more distinct lanes. Combinatorial auctions can result in more efficient allocations in multi-unit auction settings and can be promising mechanisms to allocate lanes since carriers may have substantial synergistic preferences for certain lanes and thus will bid more aggressively for the set without fear of getting an incomplete set.

Combinatorial auctions have been suggested for truckload transportation procurement and have been successfully used in several instances. Caplice and Sheffi (2003) discuss the value of combinatorial auctions in transportation procurement and present several optimization models for shippers to assign carriers to lanes. Ledyard et al. (2002), document the experience of Sears in using CA auctions for outsourcing logistics needs and report the successful use of a multi-round CA where Sears has been savings millions of dollars annually on outsourced transportation costs by using their CA mechanism. Elmaghraby and Keskinocak (2003) present the experience of Home Depot in using a single round CA mechanism for outsourcing truckload transportation capacity to move freight between the many thousands of stores of Home Depot.

One issue not discussed extensively is on normative methods for carrier bid generation in combinatorial auction settings for transportation procurement i.e. how to determine the best set of lanes. Most CA models assume that bidders know which set of lanes to bid for. In general, the evaluation of packages from the bidding perspective is difficult since there are an exponential number of possible relevant packages for a bidder. Nevertheless, bid generation is an important issue since carriers are driven by economies of scope and thus want lanes that correspond to routes that minimize vehicle reposition costs. Thus, package of lanes should be determined optimally given general information about what lanes are available.

Song and Regan (2002) present the first carrier model that uses optimization-based approximation to determine useful packages of lanes that a carrier could bid for in the context of CA for truckload transportation procurement. This work is important in that it is the first to address the bidder evaluation complexity through optimization models in the CA literature. Since there are an exponential number of possible routes it makes sense to use a normative optimization model to find the best routes. Optimization models that can be employed such as those for vehicle routing are computationally difficult to solve. However, a route obtained by using a good approximation for an appropriate optimization will represent a vast improvement over a route generated based on naive or non-optimization based methods.

The method employed by Song and Regan (2002) consists of two phases. A first phase determines heuristically the potential routes that will correspond to potentially useful packages of lanes that a carrier could bid for and then a second phase associates a binary variable with each route (package) generated in the first phase.
Each route is constructed so that all relevant operational constraints can be met. Then in a second phase a set partitioning model is constructed based on the binary variables to determine which of the packages to select. The set partitioning model is defined so that routes are selected (i.e. binary variables associated with selected routes are set to 1) subject to minimizing repositioning costs. Finally, the packages selected under the set partitioning model are submitted to the auctioneer in the form of OR bids (an OR bid is a set of packages with associated bid prices so that any subset of these bids that win would be acceptable to a carrier) see Abrache et al. (2004) and Nisan (2000) for an extensive discussion on logical bid formulations. Song and Regan (2003) also modify the above procedures to incorporate substitutable bids by creating an appropriate set cover model and bid augmentation method. They validate the carrier model in simulations where it is found that carriers that employ the optimization model benefit more than carriers that follow a simple bid selection strategy.

In this paper, we propose a carrier optimization model that integrates the generation and selection of routes. In addition, the objective of the carrier optimization model is to maximize utility which we define to be revenue from servicing a set of lanes minus the transportation costs. Given prices for lanes the model selects a package that does not necessarily achieve the least repositioning cost as defined by amount of empty movement as in Song and Regan (2003) but seeks a package that offers the most profit. The goal is to find the optimal tradeoff between the revenue provided by servicing a set of lanes and the associated repositioning costs. This is an important objective since identification of the right set of lanes will depend critically on not just repositioning costs but also on how much revenue a carrier will receive for servicing the lanes. The model is an integer program that also incorporates any existing lane commitments as well as other operational constraints. We give a decomposition approach based on column generation and Lagrangian relaxation. The main contribution of the model is in the consideration of optimal integration of route generation and selection in the presence of operational constraints under utility maximization. The model is a quadratically constrained quadratic integer program for which we develop a decomposition based heuristic. The decomposition strategy involves a partition of the model into a master and a subproblem for which a column generation-like strategy is employed to derive approximate solutions to the original carrier formulation. In addition, the model can be incorporated in multi-round settings by using approximate-price information for lanes based on tentative allocation of lanes in a round (Kwon et al., 2005). In the case of single round auctions, reservation prices for lanes can be used as coefficients in the utility maximization.

The paper is organized as follows. First, we discuss auction frameworks for truckload transportation procurement in Section 2. In Section 3 we present notations, definitions, and assumptions that pertain to the carrier optimization model that we develop in Section 4. Section 4 also gives a decomposition of the optimization model. Section 5 presents the algorithms and heuristics for solution of the optimization model in Section 4. Section 6 gives the numerical results. Section 7 addresses the model use in multiple depot situation and concluding remarks are given in Section 8.

2. Auction environment

We describe the general auction context in which a carrier would place package bids for lanes. The shipper determines which lanes to put up for bidding and then the carriers need to determine which packages of lanes to submit to the auctioneer. The auctioneer who is the shipper or an agent that represents the shipper solves an optimization problem called winner determination to decide which packages of lanes to allocate. The winner determination problem (WDP) is a NP-Hard combinatorial optimization problem see de Vries and Vohra (2003) and Rothkopf et al. (1998) for more details. Typically, the WDP is some variant of the weighted set packing problem. If a single round auction is used then no more bids are allowed after the WDP computation. For multiple rounds, bidders are allowed to submit packages again after the WDP has been computed and then another WDP computation is performed. These packages can be different from what was bid on in previous rounds depending on the auctioneer’s new or updated prices for packages, which are called ask prices. The auctioneer can list current successful prices for items or packages after a round. Various stopping rules can be used to terminate the auction see Fig. 1. There are both advantages and disadvantages in using single or multiple round auction and often depends on the particular environment.

Although a single round auction could be the context we envision the carrier model employed in a multiple round setting. The multiple round CA format could be an iterative ascending format e.g. the mechanism in
Parkes (1999) where packages prices and even individual lane prices (consistent with package prices) are available after each round. This allows for construction of a new bid that is reflective of a current allocation. Thus, each carrier need not form a complex bid such as an exclusive-or (XOR) bid since there will be opportunities to bid on different packages in future rounds and so bidders could focus on the best one at a given round. Furthermore, the iterative auction formats we consider are the type where bidders determine termination of the auctions. As long as prices are such that a bidder finds a utility maximizing (non-empty) package the auction will continue and so opportunities to bid for different packages will persist until there are no more that have positive utility in which case it would be suboptimal for a bidder who has not been allocated at termination to bid for any other package and so it is not necessary to submit other packages. If there are multiple optimal solutions to the carrier problem at a round then an XOR bid could be formed on these packages, but otherwise we assume that a single bid in submitted at a round.

Multiple round auctions have shown to be better than single round auctions in environments where bidders have hard computational requirements to evaluate worth of items see Parkes (2000). The information e.g. prices for packages obtained after a round can enable bidders to adjust their strategies and react to price changes. Since bidders involved in combinatorial auctions for transportation procurement have hard computational tasks to evaluate potential packages of lanes a multi-round format would be an ideal framework in which bidders could employ optimization to further ameliorate the computational challenges. Bidders can use current price information for packages to make adjustments in strategies. Furthermore, as shown in Kwon et al. (2005) prices for individual items in multi-round combinatorial auctions could be used as well as coefficients in bidder optimizations. For example, in the context of transportation procurement, prices for individual lanes would correspond to revenue received for servicing a lane, and so this information could be incorporated into the objective function of a carrier’s bid generation optimization to determine the best package. Prices for lanes would change during the course of an auction and so a bidder could update the objective function of her optimization to determine the best package at the current round. In general, the prices for items can be made to be reflective of a current allocation of packages as obtained through the winner determination computation with additional price (penalty) terms that act to prevent false profit opportunities in a multi-round setting see Kwon et al. (2005).

3. Notation, definitions and assumptions

Consider a transportation network, which is a graph $G=(V,E)$, where $V=\{1,2,\ldots,n\}$ are geographically dispersed nodes (cities), and $E$ is the set of edges (lanes) consisting of arcs defined by the directed arcs between each pair of nodes: one for node $j$ to $k$, and the other from $k$ to $j$. A truckload carrier has a given fixed number of trucks. A center depot is served as the home garage for all trucks the carrier owns. The carrier has already committed to provide service for a set of lanes in its transportation network and hope to add more lanes with new businesses to the network. Given the commitment to service existing lane contracts, the carrier seeks a new set of lanes that fits well with existing business with respect to repositioning and other operational costs and revenue generation. Next, we present the following common notation, definitions and assumptions.

We assume that the prices of lanes associates with new business are constant over a round. These prices are envisioned to be obtained from a tentative allocation of packages at a round as generated by a winner determination algorithm by forming an appropriate lane pricing linear program see Kwasnica et al. (2005), Kwon
et al. (2005). These lane prices are approximate marginal values that are consistent with the package allocation. For example, allocated packages are maintained to be linear in the sum of the individual lane prices and losing bid prices are such that they are too low as determined by the current lane prices. Due to the non-convexity of the winner determination method prices in general are not exact marginal prices, but suitable approximate prices are enough to guide optimizing bidders into discovery of new profitable packages. The carrier optimization problem then determines the best set of lanes based on the sum of the lane rewards minus the physical costs of the corresponding route. We acknowledge that lanes in different routes can be of different value and we account for this by having the distinct routes (packages) that a lane can be involved in to have in general different package (bid) prices reflecting the different set of lanes in the respective routes. Each route will have a different physical cost even if routes have lanes in common with each other. Thus, we do not need to have separate lanes prices corresponding to each possible route. We capture the different values of packages with lanes in common by the cost of servicing the packages as determined by the carrier model and these costs will be influenced by the other lanes not in common.

3.1. Notation and definitions

Let

\[ V \quad \text{the set of all nodes (cities),} \quad V = \{1, 2, \ldots, n\}, \]
\[ S \quad \text{one subset of all nodes} \quad V, \]
\[ \Gamma \quad \text{the set of all lanes}, \]
\[ d_{jk} \quad \text{the shortest distance between node} \quad j \quad \text{and node} \quad k, \]
\[ f_{jk} \quad \text{the existing flow volume between node} \quad j \quad \text{and node} \quad k, \]
\[ q_{jk} \quad \text{the new flow volume available for bidding between} \quad j \quad \text{and} \quad k, \]
\[ p_{jk} \quad \text{the ask price for the new flow volume} \quad q_{jk}, \]
\[ R_{jk} \quad \text{the revenue from the existing business} \quad f_{jk}, \]
\[ L_i \quad \text{the total length of the tour} \quad i, \]
\[ \omega \quad \text{the given maximum length allowed for a tour}, \]
\[ T \quad \text{the capacity or the number of all trucks the carrier has} \quad (T \geq 1), \]
\[ v \quad \text{the cost of travelling a unit distance for a truck}. \]

3.2. Assumptions

- The transportation network for each carrier is a complete graph.
- All tours start and end at the same home depot, denoted by node 1, this has significant meaning in practice: maintaining carriers’ equipment regularly at a fixed location and getting their drivers home frequently and predictably.
- All trucks are identical (in terms of capacity).
- There are integral unit demands on each lane, each truck load has one unit capacity.
- For each node \( j \), \( d_{ij} \leq \frac{\omega}{2} \) holds.
- The triangle inequality \( d_{jk} + d_{kl} \geq d_{jl} \), for \( j, k, l \in V \) and \( j \neq l \) holds.
- Each carrier is to generate a single optimal package \( S \) of lanes to submit to the auctioneer.
- The model assumes that carriers will place package bid with lanes that consist of multi-unit demand even though the truckload units can be put into different tours.

4. Problem description

4.1. The carrier’s optimal bid generation model

In transportation procurement, the carriers’ major goal is to discover and take advantage of interdependencies in their transportation operations, and then determine the optimal utility maximizing packages to bid for.
An important characteristic of transportation service is that the carriers need to consider how to reposition empty trucks and get drivers home regularly. To capture this, we identify for each lane three types at flow scenarios:

1. In type 0, the lane has \( f_{jk} \) units of flow volume due to the existing business.
2. In type 1, the lane has \( q_{jk} \) units of flow volume due to the new business.
3. In type 2, the lane has zero unit of flow volume, i.e. empty line, for the purpose of repositioning.

An overview of the carrier’s optimal bid generation model is as follows:

\[
\begin{align*}
\text{max} & \quad \text{Total utility} \\ 
\text{s.t.} & \quad \text{Each lane with existing business is covered exactly once} \\ & \quad \text{Each lane with new business is covered at most once} \\ & \quad \text{Total transportation capacity constraint} \\ & \quad \text{Other operational constraints such as: all trucks} \\ & \quad \text{must return the original depot and driving distance limitation}
\end{align*}
\]

There are five sets of decision variables defined as follows:

\[
y_i = \text{the number of times tour } i \text{ is included in the submitted packages},
\]

\[
z_{jk} = \begin{cases} 
1, & \text{if the lane from the origin } j \text{ to the destination } k \text{ is} \\
0, & \text{otherwise.}
\end{cases}
\]

\[
x_{jk}^{za} = \begin{cases} 
1, & \text{if lane } j \text{ to } k \text{ with business of type } a \text{ is in tour } i \\
0, & \text{otherwise.}
\end{cases}
\]

and

\[
z_j^i = \begin{cases} 
1, & \text{if node } j \text{ is in tour } i \\
0, & \text{otherwise.}
\end{cases}
\]

Then, the combined model for the carrier’s optimal bid generation problem CCOBG is given as

\[
\begin{align*}
\text{max} & \quad \sum_j \sum_k p_{jk} \cdot z_{jk} - \sum_i \sum_j \sum_k \sum_z v \cdot d_{jk} \cdot x_{jk}^{za} \cdot y_i \\
\text{s.t.} & \quad \sum_i y_i \leq T \\
& \quad \sum_i y_i \cdot x_{jk}^0 = f_{jk} \quad \forall j, k \\
& \quad \sum_i y_i \cdot x_{jk}^1 = z_{jk} \cdot q_{jk} \quad \forall j, k \\
& \quad \sum_z \sum_k x_{jk}^{za} = \sum_z \sum_k x_{kj}^{za} \quad \forall i, j \\
& \quad \sum_z x_{jk}^{za} \leq z_j^i \quad \forall i, j, k \\
& \quad \sum_z x_{jk}^{za} \leq z_j^i \quad \forall i, j, k \\
& \quad \sum_z \sum_k x_{kj}^{za} \geq 1 \quad \forall i \\
& \quad \sum_j \sum_k \sum_z d_{jk} \cdot x_{jk}^{za} \leq \omega \quad \forall i
\end{align*}
\]
\[ \sum_{z} \sum_{j \in S} \sum_{k \in V \setminus S} x_{jk}^{a} \geq z_{j}, \quad S \subset V, \quad 1 \in S, \quad l \in V \setminus S \forall i, \quad (2j) \]

\[ x_{jk}^{a} \in \{0, 1\} \quad \forall i, j, k, x_{i} \in \{0, 1\} \quad \forall i, j \quad (2k) \]

\[ y_{i} \geq 0 \quad \text{and integer,} \quad \forall i, z_{jk} \in \{0, 1\}, \quad \forall j, k, b_{n} \in \{0, 1\} \quad \forall n. \quad (2l) \]

The objective function (2a) maximizes the profit, in which, \( \sum_{j} \sum_{k} aP_{jk} \cdot z_{jk} \) is the total revenue due to bidding on the set of lanes, \( \sum_{j} \sum_{k} \sum_{i} x_{jk}^{a} \cdot y_{i} \) is the total travelling cost. The constraints (2b) ensure that the number of trucks needed to serve all submitted packages (tours) does not exceed the total truck capacity \( T \) of the carrier, which express the total trucks a carrier owns. The constraints (2c) make sure that the existing flow volume \( f_{jk} \) can be and must be covered exactly once in all submitted packages. The constraints (2d) ensure that the new flow volume \( q_{jk} \) can be covered at most once in all submitted packages. The constraints (2e) make sure that only pure cycles and cycles with subtours are allowed. The constraints (2f) and (2g) enforce that if one lane from node \( j \) to node \( k \) is included in the optimal solution, then both of node \( j \) and node \( k \) must be selected, which correspond to maintain the property that each lane has at most single unit of flow volume. The constraints (2h) enforce any tour to include the depot (imply \( z_{1} = 1 \) always true), or one truck must visit the depot at least once in each tour. The constraints (2i) ensure each tour has to satisfy the total length limit. The constraints (2j) eliminate those solutions that consist of subtours for which there does not exist a node \( j \) in common (node-disjointed subtours), that means, all connected subtours are allowed in this model, see Fig. 2 for more details. The constraints (2k) and (2l) are for binary variables or integral variables.

4.2. Problem decomposition

4.2.1. Decomposition framework

The carrier’s bid generation model is a non-linear integer programming, where, both the objective function and constraints (2d) are quadratic. To solve this kind of NP-hard problems is very challenging. Normally, we can solve these problems by CPLEX Mixed Integer Optimizer for MIQCP, however, the number of constraints (2k) is exponential and impossible to enumerate all of them. Thus, it is not practical to solve these problem instances directly. In addition, for the same reason it is difficult to employ the Lagrangian relaxation on those constraints (2k).

In this paper, we present a decomposition of the carrier’s optimal bid generation model into a master problem consisting of (2a)–(2d) and subproblem with the feasible region defined by (2e)–(2j) for the purpose of algorithmic development, in which all constraints of (2e)–(2j) are concerned with constructing individual tours and allows us for the proposed decomposition approach. We develop a branch and bound method to get exact solution of the master problem. In the branch and bound tree, each branching node is a linear relaxation problem for the master problem. The LP relaxation problem can be solved by column generation like decomposition approach, in which, a column represents a feasible tour that any single truck can follow as defined by the constraints of this carrier’s model. The subproblem generates feasible tours with single unit flow volume on each lane that offer the most promising improvement in utility for the master problem. The subproblem is NP-hard and solved by a heuristic first. If the heuristic yields a feasible solution with positive reduced cost (PRC), then the solution is added to the master problem as a new column; else, we solve the subproblem exactly by CPLEX.

![Fig. 2. Examples of feasible and infeasible solution for the subproblem.](image-url)
4.2.2. Carrier’s master problem

The master problem will involve two sets of decision variables: \( y_i, z_{jk} \). Furthermore, we need to use another set of decision variables \( x_{jk}^0 \) for the subproblem, and we should also mention for each given fixed column \( i \), the values of \( x_{jk}^0 \) are known constants, i.e. they are only decision variables for the subproblem, but not for the master problem.

Then, the master problem model (MP) is as follows:

\[
\begin{align*}
\text{max} & \quad \sum_j \sum_k p_{jk} \cdot z_{jk} - \sum_i \sum_j \sum_k \sum_z v \cdot d_{jk} \cdot x_{jk}^z \cdot y_i \\
\text{s.t.} & \quad \sum_i y_i \leq T \quad (\sigma) \\
& \quad \sum_i y_i \cdot x_{jk}^0 = f_{jk} \quad \forall j, k \quad (\theta_{jk}) \\
& \quad \sum_i y_i \cdot x_{jk}^1 = z_{jk} \cdot q_{jk} \quad \forall j, k \quad (\lambda_{jk}) \\
& \quad y_i \geq 0, \quad \text{and integer, } \forall i, z_{jk} \in \{0, 1\} \quad \forall j, k.
\end{align*}
\]

The constraints (3b) ensure that the number of trucks needed to serve all submitted packages (tours) does not exceed the total truck capacity \( T \) of the carrier. The constraints (3c) ensure that the existing flow volume \( f_{jk} \) can be and must be covered exactly once in all submitted packages. The constraints (3d) ensure that the new flow volume \( q_{jk} \) can be covered at most once in all submitted packages. The constraints (3e) ensure all variables integral or binary. Obviously, the MP is an integer program, and therefore NP-hard.

In fact, the real utility of the submitted packages should include another part \( \sum_j \sum_k R_{jk}, \quad \forall j, k \)

which is the total revenue from current lanes with existing flow volume. We do not take in account this part in our model because it is a constant for any feasible solution of the MP.

In this paper, we develop two different relaxation upper bounds for the master problem: Lagrangian relaxation and linear relaxation bounds. If we relax the constraints (3c), we have the following Lagrangian relaxation of the master problem (LRMP)

\[
\begin{align*}
\text{max} & \quad \sum_j \sum_k p_{jk} \cdot z_{jk} + \sum_i \sum_j \sum_k \left( \theta_{jk} \cdot x_{jk}^0 - \sum_z v \cdot d_{jk} \cdot x_{jk}^z \right) \cdot y_i - \sum_j \sum_k \theta_{jk} \cdot f_{jk} \\
\text{s.t.} & \quad \sum_i y_i \leq T \quad (\sigma) \\
& \quad \sum_i y_i \cdot x_{jk}^0 = q_{jk} \cdot z_{jk} \quad \forall j, k \quad (\lambda_{jk}) \\
& \quad y_i \geq 0, \quad \text{and integer, } \forall i, z_{jk} \in \{0, 1\} \quad \forall j, k.
\end{align*}
\]

4.2.3. Carrier’s subproblem

The dual problem (DMP) of linear relaxation of MP is given by

\[
\begin{align*}
\text{min} & \quad T \cdot \sigma + \sum_j \sum_k f_{jk} \cdot \theta_{jk} \\
\text{s.t.} & \quad -q_{jk} \cdot \lambda_{jk} \geq p_{jk} \quad \forall j, k \quad (\sigma) \\
& \quad \sigma + \sum_j \sum_k \theta_{jk} \cdot \theta_{jk} + \sum_j \sum_k x_{jk}^1 \cdot \lambda_{jk} \geq -v \cdot \sum_j \sum_k \sum_z d_{jk} \cdot x_{jk}^z, \quad \forall i \\
& \quad \sigma \geq 0, \theta_{jk}, \lambda_{jk} \text{ free, } \forall j, k.
\end{align*}
\]
For each lane $j$ to $k$, the reduced cost is defined by
\[ r_{jk} = p_{jk} + q_{jk} \cdot \lambda_{jk} \quad (6a) \]

Let a lane in a tour $i$ be expressed as lane from $j$ to $k$, then the reduced cost $r_i$ of a column $y_i$, i.e. the tour $i$ with at most single unit of flow volume, is given by
\[ r_i = -\sigma - v \cdot \sum_j x_{jk}^a \cdot d_{jk} - \sum_j x_{jk}^0 \cdot \theta_{jk} - \sum_x \sum_k x_{jk}^1 \cdot \lambda_{jk} \quad (7a) \]

Our interest is in finding a feasible tour $i$ with the maximum reduced cost $r_i$, which is a feasible column for the master problem of the carrier’s optimal bidding generation problem. Then, we develop the subproblem based on (7a). We will briefly analyze this in subsection (6.2) again. Meanwhile, we should also mention, the constraints (5c) remain the same in the dual problem of the original MP, that means, the definition of reduced cost for each tour also works for the original MP. It also provides an evidence about relaxing constraints (3e) to simplify the MP as the SMP does not change the optimal solution for the subproblem.

In this subproblem, we need another set of decision variables:
\[ z_j = \begin{cases} 
1, & \text{if node } j \text{ is in the tour,} \\
0, & \text{otherwise.}
\end{cases} \]

The subproblem is given by
\[
\begin{align*}
\max & -v \cdot \sum_j \sum_k \sum_x d_{jk} \cdot x_{jk}^a - \sum_j \sum_k x_{jk}^0 \cdot \theta_{jk} - \sum_j \sum_k x_{jk}^1 \cdot \lambda_{jk} \\
\text{s.t.} & \sum_x \sum_k x_{jk}^a = \sum_x \sum_k x_{kj}^a \quad \forall j \\
& \sum_j x_{jk}^a \leq z_k \quad \forall j, k \\
& \sum_x x_{jk}^a \leq z_j \quad \forall j, k \\
& \sum_x \sum_k x_{jk}^a \geq 1 \\
& \sum_j \sum_k \sum_x d_{jk} \cdot x_{jk}^a \leq \omega \\
& \sum_x \sum_{k \in S} \sum_{l \in S} x_{jk}^a \geq z_l, \quad S \subset V, \quad 1 \in S, \quad l \in V \setminus S \\
& x_{jk}^a \in \{0, 1\} \quad \forall j, k, a. \\
z_j \in \{0, 1\} \quad \forall j \quad (8i)
\end{align*}
\]
in which, node 1 is the depot.

The objective function value is the reduced cost for the generated tour. The constraints (8b) are for flow conservation at every node, which make sure that only pure cycles and cycles with subtours are allowed. The constraints (8c) and (8d) enforce that if one lane from node $j$ to node $k$ is included in the optimal solution, then both of node $j$ and node $k$ must be selected; furthermore, these two constraints of (8c) and (8d) are tighter than the constraints of $\sum_j x_{jk}^a \leq 1, \forall j, k$. The constraints (8e) enforce the tour must include the depot (imply $z_1 = 1$ always true), or one truck must visit the depot at least once in each tour. The constraints (8f) ensure each tour has to satisfy the total length limit. The constraints (8g) eliminate disjointed subtours. The constraints (8h) and (8i) are for binary variables. Obviously, the SP is also integer programming, and therefore NP-hard.

Fig. 2 shows one example of the feasible and infeasible solution for the subproblem respectively. This figure should be useful to help readers understand the subproblem better.
5. Algorithms and heuristics

In this section we discuss how to optimally solve the master problem and the subproblem first. A step by step description of the procedure is given below (also see Fig. 3). Then we present a heuristic to obtain a good feasible solution for the MP—the simplified branch and bound method (SBNB).

1. Generate necessary initial columns at least covering all existing businesses.
2. Solve the linear relaxation of the MP based on current columns, if the solution is integer, record it as a lower bound.

Fig. 3. The flow chart of the whole algorithm.
3. Build a subproblem based on the dual information of the MP, and heuristically solve the subproblem by Greedy algorithm or Neighborhood search algorithm, if a solution with positive reduced cost (PRC) is found, go to step 2.

4. Solve the subproblem optimally; if the optimal solution is positive, add the solution as a new column to the MP and go to step 2.

5. Start SBNB method, try to find better lower bound, more details see subsection 5.2.1.
   (a) Pick a lane according to branching rule. If there exists no node to branch out, declare the current best solution to be the optimal solution stop.
   (b) Solve the linear relaxation of the MP, if the solution is integer and better than the current best solution, record it as a lower bound.
   (c) If the solution is less than the current lower bound, return to its parent node, otherwise, go to step (a) for the next node.

6. Start the depth-first branch and bound method:
   (a) Pick a lane according to branching rule if there exists no node to branch into, declare the current best solution to be the optimal solution stop.
   (b) Solve the linear relaxation of the MP, if the solution is integer and better, record it as a lower bound.
   (c) If the solution is less than the current lower bound, return to its parent node.
   (d) When the gap between the upper bound and the current lower bound is less than 5% of the current lower bound, solve the Lagrangian relaxation of the MP, if the bound is less than the current lower bound, return to its parent node.
   (e) Based on dual information of the LP, if we can find new column with PRC by the heuristic, go to step (b).
   (f) Solve the subproblem exactly, if we can find new column with PRC, go to step (b); otherwise, go to step (a) for the next node.

5.1. Algorithms to find feasible columns for MP

There exist strong interactions between the master problem and the subproblem: the carrier can figure out more feasible tours by solving the subproblem, which are supposed to tighten the bound of the objective function value of the master problem; on the other hand, the carrier can also use the dual information of the linear relaxation of the master problem to find new tours with positive reduced cost. Next, we will discuss how to generate initial columns to form the initial master problem.

5.1.1. Naïve algorithm

No matter which set of lanes to bid for, any feasible solution must cover all existing business exactly once. The naïve way to generate a feasible solution is to only include existing business and bid for no new business. We develop a naïve algorithm to find enough feasible tours to cover all lanes with existing business. The first meaning for “naïve” is that we only generate feasible tours with those lanes serving existing business or empty lanes in this naïve algorithm. Another meaning for “naïve” is: for those lanes that serve existing business and include the depot node 1, the algorithm generates 2-lane cycles to cover them; and for those lanes that serve existing business but disjoint with the depot node 1, the algorithm generate 3-lane cycles for those lanes.

5.1.2. Greedy Algorithm

We present a “Greedy” Algorithm to find feasible solutions of the subproblem in terms of the most promising improvement in reduced cost. Define the potential improvement of lane $j$ to $k$ on reduced cost $u_{jk}$ as $u_{jk} = -\Omega_{jk} - v \cdot d_{jk}$, where, $v \cdot d_{jk}$ is the travelling cost, and

$$\Omega_{jk} = \begin{cases} 
\theta_{jk}, & \text{if lane } j \text{ to } k \text{ serving existing business, and } \theta_{jk} < 0, \ \theta_{jk} \leq \lambda_{jk}; \\
\lambda_{jk}, & \text{if lane } j \text{ to } k \text{ serving new business, and } \lambda_{jk} < 0, \ \lambda_{jk} \leq \theta_{jk}; \\
0, & \text{otherwise.}
\end{cases}$$
The algorithm works as illustrated in Fig. 4. The detail algorithm is given as follows, where \( \rightarrow \) represents an empty lane; \( \Rightarrow \) for a lane with existing business; \( \rightarrow \) for a lane with new business:

1. Set initial reduced cost \( = -\sigma \), which represents the dual value of truck capacity constraint (3b) in LP relaxation of the MP and must be included in any feasible tours. Pick the directed lane \( j \rightarrow k \) with the maximal \( u_{jk} \) and \( d_{1j} + d_{jk} + d_{k1} \leq \omega \), ties are broken by selecting the closest lane to the depot; If \( \Omega_{jk} = \theta_{jk}, j \Rightarrow k \), else if \( \Omega_{jk} = \lambda_{jk}, j \rightarrow k \). For simplification, use \( j \Rightarrow k \) to represent these two.

2. Connect node 1 with \( j \), four cases:
   - When \( j = 1, 1 \Rightarrow k \).
   - When \( u_{1j} > 0 \), and \( -\theta_{1j} - v d_{1j} > 0 \), \( 1 \Rightarrow j \Rightarrow k \).
   - When \( -\lambda_{1j} - v d_{1j} > 0 \), \( 1 \rightarrow j \Rightarrow k \).
   - When \( u_{1j} \leq 0 \), pick the node \( m \) with \( \max_m \{ -\min \{ \theta_{1m}, \lambda_{1m} \} - \min \{ \theta_{mj}, \lambda_{mj} \} - v (d_{1m} + d_{mj}) \} \), if \( m \neq 1 \), \( 1 \Rightarrow m \Rightarrow j \), else \( 1 \rightarrow j \Rightarrow k \).

3. Then connect \( k \) with 1, the similar ideas to those in step 3.
4. Calculate the reduced cost for the tour, if it is greater than zero, that is a new column, otherwise, cannot find one column with positive reduced cost, for both cases, stop.

In which, all lanes are directed, i.e. lane \( j \rightarrow k \) means a lane from start point \( j \) to the destination \( k \). Each tour found through the above searching process is a feasible solution of the MP, or can be one of initial columns for the MP if we replace \( \theta_{jk}, \lambda_{jk} \) with \( R_{jk} \) and \( p_{jk} \) respectively.

We also test other criteria in terms of the definition of \( u_{jk} \) for this algorithm, such as: \( u_{jk} = f_{jk} * d_{jk} \) if existing business with higher priority; \( u_{jk} = p_{jk}, u_{jk} = p_{jk} \lambda_{jk}/d_{jk} \) and \( u_{jk} = q_{jk} \) respectively if new business with higher priority.

5.1.3. Neighborhood search Algorithm

Neighborhood search algorithm works as a special greedy algorithm based on a concept of neighborhood. We choose a lane with the maximum \( u_{jk} \) from the neighborhood of the current node as our priority.

Define neighborhood of node \( j \) as all the nodes except to \( j \) (recall the transportation network is a complete graph). Then, the heuristic works as: first set the depot 1 as the current node. Choose the lane \( d_{1j} \) with the maximum \( u_{jk} \), if \( -\theta_{1j} - v d_{1j} > 0 \), \( 1 \Rightarrow j \Rightarrow k \), if \( -\lambda_{1j} - v d_{1j} > 0 \), \( 1 \rightarrow j \Rightarrow k \) is our beginning path; update \( j \) as the current node. then, follow the same idea till the current node reaches node 1. Finally, calculate the reduced cost. If greater than zero, the tour is a new desirable column for the MP.

5.2. The bounding techniques

5.2.1. Simplified branch and bound method

For the branch and bound method, the quality of lower bounds is critical. We know any feasible solution of the master problem is an lower bound for the MP. Unless the LP relaxation of the MP yields integral optimal
solution, we introduce a heuristic to find lower bound, which is called as SBNB method. At the root node of the branch and bound method, we implement column generation scheme and find some feasible tours, and then maintain the same set of tours throughout the whole branch and bound search to find a good feasible solution to the master problem, that means, we do not implement column generation again at each branching node to find more feasible tours with positive reduced cost within the branch and bound method. In terms of reducing the stage of implementing column generation scheme, we call this simplified heuristic as the simplified branch and bound method (SBNB) (also see Fig. 3), which may provide a reasonably good feasible solution in a relatively short time for large-size problem instances.

5.2.2. Upper bound techniques

In this subsection, we present two approaches to getting upper bound: LP bound and Lagrangian bound. Assuming that LP based bounding approaches are well known, we pay more attention to Lagrangian bound.

5.2.2.1. Lagrangian bound. In addition the LP bound, we also consider a Lagrangian relaxation upper bound that might offer an improvement over the LP bound. We relax the constraints (3c) and get the Lagrangian relaxation of the master problem (LRMP), and then employ the classical subgradient optimization method described in Fisher (1981). We briefly address how to define and initialize the key variables in the subgradient method. Given an initial multiplier \( \theta_{jk}^0, \forall j, k \), a sequence of multipliers is generated by

\[
\theta_{jk}^{n+1} = \max \left\{ 0, \theta_{jk}^n + \ell^n \left( \sum_i y_i \cdot x_{jk}^i - f_{jk} \right) \right\}
\]

where \( \ell^n \) is a positive scalar step size defined as follows:

\[
\ell^n = \frac{u^n \left( z_{LR}(\theta_{jk}^n) - \zeta z_j \right)}{\sum_k \sum_i (\sum_i y_i \cdot x_{jk}^i - f_{jk})}
\]

where, \( z_{LR}(\theta_{jk}^n) \) is the optimal solution to the problem LRMP(\( \theta_{jk}^n \)) and \( u^n \) is a scalar satisfying \( 0 \leq u^n \leq 2 \); \( \zeta z_j \) is the current best lower bound (current best feasible solution). We start with \( u^n = 2 \) and cut it by half every time \( z_{LR}(\theta_{jk}^n) \) fails to increase after a fixed number of iterations. The method is terminated either when the upper and lower bound are sufficiently close to each other or when the iteration limit is reached.

5.3. Optimal solution strategy for the MP and subproblem

We now present the depth-first branch and bound method to solve the MP. We solve the linear relaxation of the MP first. If it yields an integer solution, then we have an optimal solution, else, we start the branch and bound algorithm.

Either a linear or a Lagrangian relaxation solution provides an upper bound for the MP, which also automatically offers an upper bound for the combined carrier’s optimal bid generation model (CCOBG). On the other hand, any feasible solution to the MP provides a lower bound. There are also two ways to get feasible solutions. First, the linear relaxation of the MP happens to have integer solutions. Second, we can solve the MP directly by Integer programming solver to get an integral solution. The branching rule is pretty simple: pick the lane \((j, k)\) with the value \( z_{jk} \) having the maximum integer infeasibility, i.e., \( z_{jk} = \arg\min_{z_{mn}} \{ |z_{mn} - 0.5| \} \).

If the heuristic fails to find any feasible solution with positive reduced cost, we have to solve the subproblem exactly. It is, however, virtually impossible to enumerate exponentially many constraints to eliminate disjointed subtours. This is the motivation why we decompose the SP into multiple easier problems. After dropping constraint \((8g)\) from SP, We call the remaining problem as remaining SP (RSP). Then we use a two-step method to solve the SP. First, we solve the RSP to get a possibly feasible solution to the SP. To see if it is indeed feasible to the SP, we check the solution with constraints \((8g)\). If the solution does not satisfy constraint \((8g)\), we block the current solution and solve the RSP again. This approach is more efficient in computation due to the reduced size of the RSP compared with the SP.
5.4. Worked example

In this subsection, we present a worked example of the carriers’ optimal bid generation model. In this example, there are three nodes, two lanes with existing business, and three lanes with new business (see Fig. 5). Each lane is a unit distance long and node one represents the only depot. Travelling a unit distance incurs fixed transportation cost of one to all the carriers regardless of the amount of load on the truck. The maximal length of each tour is limited by four units. The carrier has 180 trucks and all trucks are identical. The current ask price matrix for each lane with new business is given as Table 1.

The example can be easily solved as

1. Generate initial feasible tours such as: \( y_1: 1 \Rightarrow 3 \Rightarrow 1, \ y_2: 1 \Rightarrow 3 \Rightarrow 1, \ y_3: 1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1, \ y_4: 1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1, \ y_5: 1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1, \ y_6: 1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1. \)
2. Form the initial MP and solve the linear relaxation of the MP.
3. Based on the dual information of the linear relaxation of the initial MP, generate a new feasible tour with positive reduced cost by either Greedy algorithm or Neighborhood search method (both works): \( y_7: 1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1. \)
4. Update the MP and solve its linear relaxation.
5. Repeat step 3 and step 4, find 3 new feasible tour with positive reduced cost by Greedy algorithm and Neighborhood search method at each iteration respectively: \( y_8: 1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1, \ y_9: 1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 2 \Rightarrow 1, \) and \( y_{10}: 1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 1. \)
6. Because we can not find feasible tour with positive reduced cost by any algorithms at the fifth iterative, we solve the subproblem exactly.
7. After solving the subproblem optimally and finding that no feasible tour with positive reduced cost exists, we know the current solution is the optimal solution for the MP: \( y_1 = 20, \ y_3 = 60, \ y_7 = 40, \ y_9 = y_{10} = 20, \) and all of the rest zero.
8. The optimal bid: the set of lanes 1 \( \Rightarrow 2, \ 3 \Rightarrow 2, \ 3 \Rightarrow 1 \) at price of \( 170 + 240 + 180 = 590. \)

By the way, the carrier will get the profit of 70 with the empty repositioning cost of 60 in the optimal solution. If the carrier chose to minimize the empty repositioning cost as the objective, then, the final result should

\[ \text{Table 1} \]

<table>
<thead>
<tr>
<th>Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>–</td>
<td>170</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>240</td>
<td>180</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 5. Flow information in the worked example.
be to bid for the set of lanes 3 → 2, 3 → 1 at price of 240 + 180 = 420 with the empty repositioning cost of 40, but only with the profit of 0.

6. Computational experiments

In this section, we report our computational experiments for the carrier’s optimal bidding generation problem. All the proposed algorithms are implemented in C and run on a personal computer with a Pentium 4 2.4 GHz CPU. We use CPLEX (version 9.0) to solve all linear relaxations of the MP and the subproblems.

6.1. Configuration of test problems

We generate distance matrix, matrix for existing flows, matrix for new flows, ask price matrix, truck capacity, and limit on the length of tours as follows:

- Distance matrix $d_{jk}$ derived from actual distances between major cities in North American;
- The existing flow volume matrix $f_{jk}$, and new flow volume matrix $q_{jk}$ are randomly generated according to a uniform distribution in a range of $[0, 60]$. The ask price matrix $p_{jk}$ is proportional to the product of flow volume and distance, in which, the ratio are uniformly distributed in range of $[1, 2]$;
- For problem instances with 6, 8, 10, 12, 15, 17, and 20 nodes, the average numbers of lanes with new business and existing business are showed in Table 2;
- The truck capacity is determined randomly and ranged between 50% and 100% of the sum of all existing flow volumes;
- The limit on the total length of tours is the product of the number of working hours in each unit time period and the average travelling speed per hour, i.e., the number of working hours set at 40 h/week, the average travelling speed at 50 miles/h. For large examples with nodes more than or equal to 17, the limit on the total length of tours is a little more than the above in order to be long enough to cover all lanes in this network.
- Each pair of Big and small problem instances are derived from a same problem instance. The big instances have more exiting flows and larger transportation capacity, whereas the small ones have less flows and smaller capacity. With other aspect of the problem, both share the same parameters.

6.2. Computational results

Table 3 presents the main computational results, where, the first number in each name reflects the number of nodes while the second of the index of the problem instance with the same number of nodes. For instance, 6-1 means the first example with 6 nodes, 6-2 the second example with 6 nodes, 6B the example with 6 nodes and Big problem, 6S the example with 6 nodes and Small problem, and so on. "SBNB solutions" mean the feasible solutions provided by simplified branch and bound method (SBNB). Both “Final (w/o)” and “Final (w)” are the running times to obtain the same final optimal solutions for the MP by the branch and bound method with column generation, the only difference is to obtain “Final (w)” results by solving SBNB to get initial lower bound in the Branch and Bound process, but “Final (w/o)” without using SBNB. All results

Table 2
The average number of lanes with new and existing business

<table>
<thead>
<tr>
<th>Example (nodes)</th>
<th>No. of lanes with new business</th>
<th>No. of lanes with existing business</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>35</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>48</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>76</td>
<td>28</td>
</tr>
<tr>
<td>15</td>
<td>131</td>
<td>42</td>
</tr>
<tr>
<td>17</td>
<td>210</td>
<td>52</td>
</tr>
<tr>
<td>20</td>
<td>335</td>
<td>64</td>
</tr>
</tbody>
</table>
in the 2nd, 3rd, 4th columns are CPU times in seconds. “Gap” represents a measure about the quality of SBNB solution. We use “Solution with existing business” to represent the profit a carrier to bid no new business, then “SBNB solution—solution with existing business” can be viewed as the profit improvement generated by the SBNB method, and “Optimal solution - Solution with existing business” as the maximal profit improvement the carrier can obtain by getting the final optimal solution.

\[
\text{Gap} = \frac{\text{SBNB solution} - \text{solution with existing business}}{\text{Optimal solution} - \text{solution with existing business}}
\]

in which, solution with existing business is the optimal solution only based on existing business, or, submitting no new business for that carrier in this auction.

From Table 3, we make the following observations and conclusions:

- The method without SBNB works well for relatively small problem instances, say, with the number of nodes less than or equal to 10; the method with SBNB provides optimal solutions faster for relatively large problem instances such as instances with 10 or more nodes.
- Problems (6B, 8B, 10B, 12B) with more exiting flows and larger transportation capacity takes less time to be solved than those (6S, 8S, 10S, 12S) with less flows and smaller capacity even when the number of nodes in such problem instances is the same.
- The gaps in Table 3 suggest SBNB may provide the carrier a good approximation to the optimal solution within a relatively short computation time for the large-size instances.
- In most cases the linear relaxation of the MP at the root node of the branch and bound tree yields integer solutions, hinting that the MP may have the integer-friendly programming property (Re Velle, 1993). However, it becomes less often to have integer solutions as more columns are added. We guess the reason may be that it is more difficult to find optimal solution for a given set of columns (or tours) as more columns are included in MP. However, the resulting solution (optimal solution for a given set of columns) must be closer to the ultimate optimal solution (optimal solution with all tours considered).

<table>
<thead>
<tr>
<th>Example</th>
<th>Init by SB&amp;B</th>
<th>Final (w/o)</th>
<th>Final (w)</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-1</td>
<td>6.97</td>
<td>1.66</td>
<td>7.48</td>
<td>6.92</td>
</tr>
<tr>
<td>6-2</td>
<td>0.05</td>
<td>24.34</td>
<td>23.84</td>
<td>23.84</td>
</tr>
<tr>
<td>6-3</td>
<td>0.50</td>
<td>13.91</td>
<td>13.08</td>
<td>0</td>
</tr>
<tr>
<td>6-4</td>
<td>0.64</td>
<td>69.63</td>
<td>50.64</td>
<td>2.89</td>
</tr>
<tr>
<td>6B</td>
<td>0.53</td>
<td>0.67</td>
<td>0.72</td>
<td>0</td>
</tr>
<tr>
<td>6S</td>
<td>6.19</td>
<td>1.33</td>
<td>7.33</td>
<td>30.19</td>
</tr>
<tr>
<td>8-1</td>
<td>26.27</td>
<td>2.88</td>
<td>28.20</td>
<td>13.05</td>
</tr>
<tr>
<td>8-2</td>
<td>70.81</td>
<td>42.48</td>
<td>97.08</td>
<td>2.62</td>
</tr>
<tr>
<td>8-3</td>
<td>59.22</td>
<td>75.44</td>
<td>119.89</td>
<td>3.53</td>
</tr>
<tr>
<td>8-4</td>
<td>87.00</td>
<td>92.44</td>
<td>186.25</td>
<td>7.80</td>
</tr>
<tr>
<td>8B</td>
<td>4.86</td>
<td>3.03</td>
<td>6.05</td>
<td>0</td>
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<tr>
<td>8S</td>
<td>99.08</td>
<td>1560.17</td>
<td>2609.64</td>
<td>18.74</td>
</tr>
<tr>
<td>10-1</td>
<td>907.36</td>
<td>1928.03</td>
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<td>2.84</td>
</tr>
<tr>
<td>10-2</td>
<td>126.72</td>
<td>3271.53</td>
<td>2883.22</td>
<td>0.89</td>
</tr>
<tr>
<td>10-3</td>
<td>26.34</td>
<td>1704.97</td>
<td>3779.41</td>
<td>5.70</td>
</tr>
<tr>
<td>10-4</td>
<td>98.67</td>
<td>1175.39</td>
<td>2869.59</td>
<td>1.72</td>
</tr>
<tr>
<td>10B</td>
<td>29.06</td>
<td>74.20</td>
<td>184.95</td>
<td>0</td>
</tr>
<tr>
<td>10S</td>
<td>421.33</td>
<td>3498.50</td>
<td>6026.11</td>
<td>0.02</td>
</tr>
<tr>
<td>12-1</td>
<td>2222.02</td>
<td>13443.75</td>
<td>7695.20</td>
<td>0.35</td>
</tr>
<tr>
<td>12-2</td>
<td>485.59</td>
<td>11814.63</td>
<td>11393.91</td>
<td>0.79</td>
</tr>
<tr>
<td>12-S</td>
<td>5334.41</td>
<td>16139.16</td>
<td>11573.67</td>
<td>1.89</td>
</tr>
<tr>
<td>15</td>
<td>2805.20</td>
<td>62278.80</td>
<td>40518.74</td>
<td>3.80</td>
</tr>
<tr>
<td>17-1</td>
<td>1928.42</td>
<td>*</td>
<td>15171.95</td>
<td>9.45</td>
</tr>
<tr>
<td>17-2</td>
<td>1763.72</td>
<td>*</td>
<td>250176.20</td>
<td>10.79</td>
</tr>
<tr>
<td>20</td>
<td>4514.67</td>
<td>*</td>
<td>432746.28</td>
<td>32.84</td>
</tr>
</tbody>
</table>

Note: * unsolved completely, due to too long times.
For the MP, there are three Lagrangian relaxation alternatives: relaxing the capacity constraint of (3b), the existing business flow constraints of (3c), or the new business flow constraints of (3d), respectively. We have experienced the best performance when constraints (3c) were relaxed and all the computational experiments have been done with these constraints relaxed.

According to Fisher (1981), Lagrangian relaxation bounds (LR) should be tighter than linear relaxation bounds (LP). However, it is difficult and not guaranteed to always get the optimal Lagrangian relaxation solution. We have found that the Lagrangian relaxation provides better bounds than the linear relaxation in roughly 20% of all the cases that we had tested within a certain time period or a certain number of iterations. It is often the case that doing more computation to tighten the LR bound does not turn out to be profitable in terms of the overall performance due to rapidly increasing computation for better LR bounds.

Recall we discuss the reduced cost for each lane and the reduced cost for each tour in subsection 4.2.3. From the constraints (5b), it is natural to conclude the bigger the value of \( r_{jk} \), the higher priority. We tried to generate new feasible columns based on \( r_{jk} \), which provides few new columns with positive reduce cost for the MP, and never finds any new columns after the programs enter the branch and bound method phase. It seems only to work occasionally at the first beginning of generating feasible columns to form the MP.

Besides the SBNB method to get feasible solution and lower bound for the carrier’s optimal bid, we also try another method for those examples with more than or equal to 15 nodes: solving the whole problem within in a certain time. For instance, we set the time limit as 120 min, and then treat the best solution found within that time period as an approximation of the final optimal solution. Our method can be expressed as follows.

1. Get two first feasible solutions.
2. If the total time exceeds 2 h limit, we stop, otherwise, go to the next step.
3. If the improvement of the latest solution from the previous one is larger than 5%, then we solve further and get another better feasible solution, then go to Step 2; otherwise, we stop.

The related results are given as Table 4. From this table, our method can provide us a good approximation to the optimal solution.

### 7. Carrier model use in multiple depot situations

In this section, we briefly discuss how the carrier model can be used in the context of a decision support system for carriers when they have more than one depot that can be used to service lanes. The carrier optimization model can be applied to each distinct depot. Each depot has its own capacity levels e.g. number

<table>
<thead>
<tr>
<th>Example</th>
<th>CPU time for optimal solution</th>
<th>Time point</th>
<th>Quality of solution (to optimal solution) (%)</th>
</tr>
</thead>
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<tr>
<td>15</td>
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<td>40.80 s</td>
<td>91.00</td>
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<td>59.98 s</td>
<td>94.53</td>
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<td>81.98 s</td>
<td>94.82</td>
</tr>
<tr>
<td>17-1</td>
<td>15171.95 s (4.214 h)</td>
<td>1928.42 s (0.536 h)</td>
<td>90.55</td>
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<td>2152.89 s (0.598 h)</td>
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<td>2289.66 s (0.636 h)</td>
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<tr>
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<td>864.58 s (0.240 h)</td>
<td>79.64</td>
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<td>3539.72 s (0.983 h)</td>
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<td>14406.78 s (4.000 h)</td>
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<tr>
<td>20</td>
<td>432746.28 s (120.207 h)</td>
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<td>73350.69 s (20.375 h)</td>
<td>99.98</td>
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Table 4
The intermediate results for big examples
of trucks and drivers and operational requirements or restrictions that can be reflected in the optimization model. The lanes that each depot could bid for may be pre-determined by the carrier organization so that the depots belonging to the same carrier do not compete with each other. Or the depots could contend for the same lanes. Each carrier solves their unique bid generation model and the carrier could amalgamate all of the bids from the different depots into an OR bid. The winner determination problem would ensure that the same lane never gets allocated more than once. Since each submitted bid would give non-negative utility carriers gains if any of the submitted bid wins.

An alternative approach would be to model the carrier’s bid generation problem with multiple depots in a single formulation. The computational complexity of would be greatly increased. The decomposition strategy may offer a much more tractable and reasonable alternative however.

8. Conclusion and future research

Song and Regan (2003) have validated the use of optimization models for carriers in combinatorial auctions for transportation procurement. The contribution of this paper lies in the development of a model that integrates route (package) generation and selection simultaneously, which has never been considered before. In addition, we present a column generation approach to solve the underlying non-linear quadratic integer program which is a unique approach for this class of optimization models. The model represents a utility maximizing decision problem that carriers can use to determine the best packages for bidding in a combinatorial auction. The model trades off revenue from servicing a set of lanes and the physical costs or repositioning. The algorithms that we develop for the carrier model can handle scenarios involving hundreds of lanes.

We have considered instances for carrier models with up to 335 lanes and the decomposition algorithm presented effectively computes optimal solutions up to 200 lanes, and we envision the carrier model to be only part of the optimization for the carrier’s entire network. That is, we envision that the carrier model in this paper pertains to a single node (depot) of the carrier. We associate a carrier optimization with a depot to capture realistic features of driver and equipment re-origination (i.e. both often must be returned to the same physical location especially the drivers). If there are more depots then the optimization formulation is used separately for each depot, each depot problem would be responsible for lanes around a certain distance away from the depot which is realistic in practice. If a carrier were capable of servicing thousands of lanes then the there would be a decomposition of lane responsibility per depot. In this manner, tractability can be attained for situations involving thousands of lanes.

Combinatorial auctions for transportation procurement are very promising frameworks for procurement of lanes having advantages for both the shipper and carriers. In general, developing models for carriers in this context is an important activity and our future research includes validating the carrier model developed in this paper in a multi-round setting employing single item price information as in Kwon et al. (2005). Another important research direction is in the incorporation of win/loss probabilities for lanes and including the uncertainty in the actual volume that will realize for a given lane. Both of these aspects are important issues in real world TL combinatorial auctions. In particular, approximation dual (price) information can be obtained from WDP solution and be used as coefficient \( p_{jk} \) for each round of the auction.

References


