A Multistage Formulation for GENCOs in a Multi-Auction Electricity Market[1]

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Abstract

In this paper we deal with the definition of a decisional model for a producer operating in a multi-auction electricity market. The decisions to be taken concern the commitment of the generation plants and the quantity of energy required to offer to each auction and to cover the bilateral contracts. We propose a Multistage Stochastic Programming model in which the randomness of the clearing prices is represented by means of a scenario tree. The risk is modelled using a Conditional Value at Risk term in the objective function. Experimental results are reported to show the validity of our model and to discuss the influence of the risk parameters on the optimal value.
Keywords: Electricity Market, Multistage Stochastic Programming, Bidding Strategy, Conditional Value at Risk.

1 Introduction

In the last few years the liberalization process that has spread over many electricity markets around the world has generated deep changes in an economic context that has been very conservative for a long time. The electricity industry is evolving into a distributed and competitive framework in which market forces drive the price of electricity both on the buying and the selling sides. The main difference with respect to the previous structure has been the introduction of competition into the different phases that characterize the electricity system.

In this new context the operators have to face new operational problems for the efficient management of their activities, since new issues, such as, for example, the market price forecasting and the risk management, have become critical ([1]). Power generation planning and operation that explicitly include both the randomness and dynamics of electricity markets into a mathematical model is already a consolidated approach in the scientific literature. Earlier works have focused on the use of the two-stage stochastic framework to incorporate the randomness into the mathematical models of well known problems such as the unit-commitment, the capacity expansion, and the bidding strategy definition ([2], [3], [4]). However, the two-stage framework succeeds to capture efficiently the stochastic aspect but only partially the dynamics of the problem because of the repetitive nature of

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the electricity market. A first improvement has been done by developing two-stage multi-period models, but afterwards the scientific community has realized that the multistage framework is becoming the most appropriate framework to capture both the dynamic and stochastic aspects of the power planning problems. Several contributions on the definition of effective decision tools for the market operators have been already proposed. Among these we cite the work of Hochreiter et al. that deals with the solution of the problem of big consumer portfolio definition ([5]). The problem of structuring portfolios of bilateral energy trading contracts has been discussed in [6] and a multistage stochastic mixed 0/1 model for its solution has been presented. In [7] a methodology for the strategic bidding problem in the case of a price-maker hydro agent with multiple plants based on a multistage programming approach has been proposed. The problem of scheduling of wind power generation in a detailed representation of the grid operation has been discussed in [8] and the problem has been formulated as a three-stage stochastic problem.

Multistage mixed-integer models for the power scheduling in hydro-thermal systems have been proposed in [9] and [10] and solution methods based on Lagrangian relaxation have been developed. Earlier, Takriti et al. have solved the unit commitment problem by means of a multistage stochastic model when load demand is uncertain ([11]).

In such a complex context, most of the contributions consider the possibility of trading in a single electricity auction ignoring the opportunity of operating in multi-auction markets. (A detailed description of the multi-auction structure is reported in section 2.) Little attention, indeed, has been paid to the multistage stochastic power scheduling in a multi-auction and/or multi-market environment. In this context we are aware of two different contributions. The first one, due to Plazas et al., defines a bidding strategy for a producer participating in a sequence of three spot markets [12]. The state of the generations units is considered to be known in advance except for the units dedicated to the AGC market. The resulting multistage model is made computationally attractable through a scenario reduction approach and solved by using general purpose software package. The second contribution, due to Triki et al., proposes a capacity allocation approach by means of which the GENeration COmpany (GENCO, for short) can decide not only the quantities to offer to each of the available auctions but also the commitment of each unit generation [13]. Their mathematical model results to be a multistage stochastic nonlinear program and a general purpose solver has been used for its solution as well.

This paper can be considered as an extension to the contribution of this last reference. However, while the formulation proposed in [13] deals just with the capacity allocation problem this work broadens the interest to the definition of a bidding strategy. The new features aim at proposing a more realistic and general representation of the decisional process allowing the GENCO to maximize its profits and at the same time to monitor the risk due to the market operations. More specifically, with respect to [13], the following critical issues have been added to the decisional process:

- ensure the respect of previously committed bilateral contracts in the energy balance;

- introduce a more accurate representation of the production units' dynamics and costs;

- define a units' generation schedule for the auctions offers and bilateral contracts energy requirements by means of specific selling bids on the Day Ahead Market;
- include the possibility of buying energy on the Adjustment Market in order to "correct" undesirable Day Ahead Market outcomes;
- consider and model explicitly the zonal market paradigm;
- incorporate a risk aversion tool of the GENCO by means of a modern risk measure like the Conditional Value at Risk (CVaR).

The remind of the paper is organized as follows. Section 2 contains an overview of the main issues of multi-auction electricity markets. Section 3 reports a more detailed description of the decisional problem. In particular, a multistage mathematical model is presented and some critical issues, like the risk management, are discussed. Section 4 reports the computational experiments that we have carried out in order to validate the effectiveness of the proposed decisional approach. Some concluding remarks end the paper.

2 Market structure

In many countries, like Italy for example, the new competitive paradigm provides two ways for the GENCOs to sell electric energy: (i) bilateral contracts, that are independent agreements between producers and eligible consumers, and (ii) power pool, i.e. an e-commerce marketplace articulated in several consecutive sessions where producers and consumers submit production and consumption bids, respectively. More specifically, the power pool may be organized in three different sessions, each with its peculiar auction mechanism:

- Day-Ahead Market (DAM) for the wholesale trading of energy between producers (GENCOs) and wholesale customers. This market usually takes place in the morning of the day ahead of the delivery day;

- Adjustment Market (AM), where market participants may revise the schedules resulting from the Day-Ahead Market, by submitting additional demand bids or supply offers. This market takes place immediately after the Day-Ahead Market, usually in the afternoon;

- Ancillary Services Market (ASM), where market participants submit offers/bids to increase or decrease injection or withdrawal for each elementary time period. The market grid operator uses these offers to correct the schedules which violate the transmission limits on the grid and to create reserve margins for the following day, or to balance in real-time the system in case of deviations from the schedules. Based on the jurisdiction to be considered, these services may be organized in only one or in different sessions.

Moreover, it is worthwhile noting that at the end of each offer/bid submission deadline, the market operator activates the market solution process. For each hour of the following day, the market algorithm accepts offers/bids in order to maximize the value of the transactions (economic merit criterion), while satisfying the transmission limits between zones (see for example [14]).

In other terms, the market operates with a zonal model, which has been successfully tested in many European countries as well as in almost all the liberalized markets of the United States and Oceania. If at least one transmission limit is violated, then the algorithm will "split" the market into two market zones: one export zone, including all the zones lying above the constraint, and one import zone, including all the zones lying below the constraint. This crossing process is repeated in each zone, building a supply curve for each market zone (including all supply offers submitted in the same zone, as well as the maximum imported quantity) and a demand curve (including all demand bids submitted in the same zone, as well as a quantity equal to the maximum exported quantity). The result will be a zonal clearing price, P_z, different in each zone z, at which all the supply offers referred to such zone will be valued. In particular, P_z will be higher in import zones and lower in export ones. If, as a result of this solution, additional constraints are violated, then the market splitting process will be replicated within the already created zones, until the market result satisfies the grid constraints. This paradigm reflects an hybrid model of market structure that offers a true customer choice and encourages the creation of a wide variety of services and price options to best meet individual customer needs. An overview on the liberalized electric market features is reported in [15], whereas a detailed description of the Italian market structure can be found in [16].

3 Problem formulation

The problem we are facing is related to the definition of the optimal quantities to be bought/sold by a GENCO operating in a multi-auction market like the one described in section 2. Moreover, we have assumed that the seller is a price-taker, i.e. with no possibility to exercise market power to affect the clearing price. This assumption is realistic for small sellers and without collusion and barriers entry into the market. Moreover, we have not considered the definition of a supply function with complex combinations of prices and quantities. We simply assume that the GENCO offers a price that is a user-defined fraction of the foreseen clearing price in order to have a confidence level on the bid acceptance. In order to simplify the discussion we consider just the real-time ASM, so to have three consecutive market sessions. The extension to include other eventual sessions is straightforward and simply involves additional variables with the corresponding constraints but presents no further complexities.

A GENCO that operates in such a complex market with the aim of maximizing its profits should determine the optimal bidding strategy and capacity allocation both at single unit level and corporate level, taking into account several issues; in addition to the restrictions imposed by the units' physical/operational limitations and market regulations, a GENCO may want to include strategic objectives such as diversification and market niche. The complexity of this decisional process increases with the number of units and also as more markets are considered but increases principally if the uncertainty is explicitly

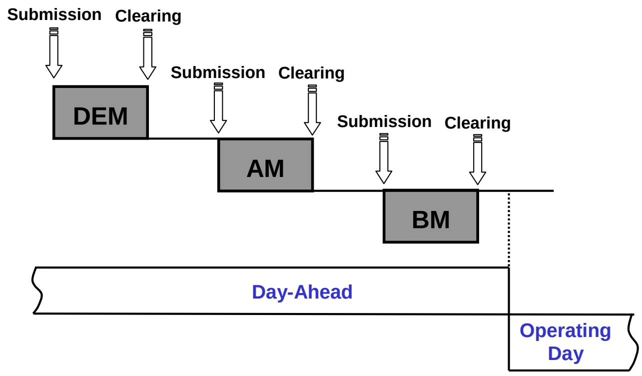

Figure 1: Sequence of decisions

taken into account. Some of the most important data are inherently uncertain: at the beginning of every auction the clearing price and quantities bought/sold are not known.

The aim of this work is the definition of a mathematical model to support the decisional process of a GENCO that wants to allocate its production capacity in the most profitable way but respecting a certain level of risk aversion.

As introduced above, this decisional process is dynamic, since there are many decisional phases corresponding to the multi-auction framework, as depicted in figure 1. Moreover, the decisional process is also made under uncertainty, since the amounts of energy effectively sold, and the clearing prices, depend on the market auction outcomes. An effective decisional approach cannot ignore this uncertainty but, on the contrary, should take into account all the random events that may occur.

These two main characteristics have suggested the adoption of Multistage Stochastic Programming as a modelling framework for this problem. In order to represent the uncertainty, i.e. the possible realizations of clearing prices, we have adopted a scenario tree representation (see figure 2). The root node stands for the first stage and corresponds to the immediately observable deterministic data. The nodes in successive stages correspond to possible outcomes of the various market sessions and each one is characterized by a certain probability of occurrence. Moreover, each node k, except the root node, has a unique predecessor $p(k)$ in the preceding stage and a finite number of successors in the next stage. Nodes without successors are the *leaves* of the tree and correspond to the scenarios: a scenario can be seen, thus, as a path from the root node to a leaf and represents a joint outcomes of the problem uncertain data over all the market sessions.

The scenario tree is also associated with the sequential decisional process, so that to each node corresponds a decision variable that depends not only on the previous decisions but also on the outcomes of the random data so far observed as well.

3.1 Problem data

In this section we briefly introduce the problem parameters starting from the (deterministic) generators characteristics and ending with the (uncertain) market outcomes.

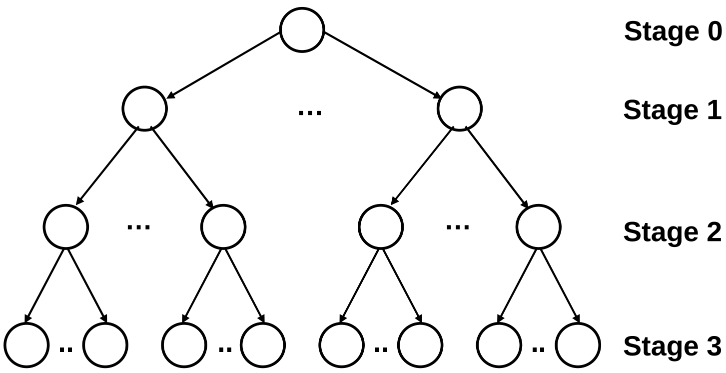

Figure 2: Scenario tree representation

- T planning horizon, usually one day divided into 24 elementary hourly period;
- I set of available production units, each one located into a particular market zone;
- Q_i^{min}, Q_i^{max} minimum and maximum capacity of thermal unit i (MWh);
- α_i, β_i fixed and linear component of the variable cost of unit i (Euro and Euro/MWh, respectively);
- SU_i, SD_i fixed start-up and shut-down cost of unit i (Euro);
- UT_i, DT_i minimum up- and down-time of unit i (hours);
- Up_{i0}, $Down_{i0}$ number of time periods unit i has been online or off-line at the beginning of the planning horizon (hours);
- U_{i0} initial status of unit i (1 if it is online, 0 otherwise);
- Q_t^{bil} total quantity of energy comitted for bilateral contracts for period t (MWh);
- S set of likely outcomes (*scenarios*) of the DAM;
- η^s occurrence probability of scenario $s \in S$;
- λ_{it}^s DAM zonal price at period t, under scenario s for unit i (depending on the zone unit i belongs to) (Euro/MWh);
- γ_{it}^s percentage of bid acceptance in the DAM for unit i, in period t under scenario s;
- L set of likely outcomes (*scenarios*) of the AM;
- π^l occurrence probability of scenario $l \in L$;
- μ_{it}^l AM zonal price at period t, under scenario l for unit i (depending on the zone unit i belongs to) (Euro/MWh);

- ν_t^l AM clearing price at period t, under scenario l (unique for all the maket zones) (Euro/MWh);
- δ_{it}^{l+} percentage of selling bid acceptance in the AM for unit i, in period t under scenario l;
- δ_{it}^{l-} percentage of purchasing bid acceptance in the AM for unit i, in period t under scenario l;
- V set of likely outcomes (*scenarios*) of the ASM;
- θ^v occurrence probability of scenario $v \in V$;
- ς_t^v ASM clearing price at period t, under scenario v (Euro/MWh);
- ρ_{it}^v percentage of bid acceptance in the ASM for unit i, in period t under scenario v.

3.2 Decisional variables

The model variables represent the decisions that the GENCO has to take in order to plan its bidding strategy and, according with the market outcomes, the production schedule. More specifically, the GENCO has to define which first units to commit for each time period and the quantities to offer in each market sessions. We indicate with x_{it} the quantity of output of unit i to offer in the DAM for time period t. Moreover, the production decisions must take into account the presence of bilateral contracts stipulated time before the planning horizon. The quantity of energy to satisfy this demand for each time period t is represented by means of one or more offers on the DAM at price 0 (x_{it}^{bil}). This issue introduces another decisional level for the GENCO who has to decide which unit (or units) each contract has to be referred to. In terms of multistage notation, all the variables x_{it} and x_{it}^{bil} are first stage decisions. On the basis of each DAM outcome s, the GENCO could correct its offers, with both selling and buying bids. We indicate with y_{it}^{s+} the quantity of energy of the selling bid on the AM for the unit i for time period t under scenario s. Similarly, y_{it}^{s-} represents the buying bid on the AM related to unit i for time period t, under scenario s. Moreover, the GENCO can offer for each unit i and for each time period t a quantity of energy on the ASM z_{it}^l, under scenario l that is a particular outcome of AM. Finally, according to the observed outcome of the ASM v, the energy quantity to be produced (Q_{it}^v) is defined. We choose to consider as first-stage decisions the state variables U_{it}, according to a realistic view of the operational profile definition for the generators. In order to represent the start-up and the shut-down state of each unit i from time period $t-1$ to time period t we introduce the binary variables Δ_{it}^+ and Δ_{it}^-, that are strictly linked to the state variables U_{it}, as it will be explained later.

3.3 Model constraints

Most of the model constraints are similar to those proposed in the work of Triki et al. ([13]), but they take into account a larger set of operational restrictions and the presence of new decisional variables:

$$
\begin{aligned}
& x_{it} + x_{it}^{bil} \leq Q_i^{max} U_{it} && \forall i, \forall t && (1)\\
& \sum_{i=1}^{I} x_{it}^{bil} = Q_t^{bil} && \forall t && (2)\\
& y_{it}^{s+} \leq Q_i^{max} U_{it} - x_{it}^{bil} - \gamma_{it}^{s} x_{it} && \forall i, \forall t, \forall s && (3)\\
& y_{it}^{s-} \leq \gamma_{it}^{s} x_{it} && \forall i, \forall t, \forall s && (4)\\
& y_{it}^{s+} \leq M \varphi_{it}^{s+} && \forall i, \forall t, \forall s && (5)\\
& y_{it}^{s-} \leq M \varphi_{it}^{s-} && \forall i, \forall t, \forall s && (6)\\
& \varphi_{it}^{s+} + \varphi_{it}^{s-} \leq 1 && \forall i, \forall t, \forall s && (7)\\
& z_{it}^{l} \leq Q_i^{max} U_{it} - x_{it}^{bil} - \gamma_{it}^{p(l)} x_{it} - \delta_{it}^{l+} y_{it}^{p(l)+} + \delta_{it}^{l-} y_{it}^{p(l)-} && \forall i, \forall t, \forall l && (8)\\
& Q_{it}^{v} = x_{it}^{bil} + \gamma_{it}^{p(p(v))} x_{it} + \delta_{it}^{p(v)+} y_{it}^{p(p(v))+} - \delta_{it}^{p(v)-} y_{it}^{p(p(v))-} + \rho_{it}^{v} z_{it}^{p(v)} && \forall i, \forall t, \forall v && (9)\\
& Q_{it}^{v} \geq Q_i^{min} U_{it} && \forall i, \forall t, \forall v && (10)\\
& \sum_{t=1}^{G_i} (1 - U_{it}) = 0 && \forall i && (11)\\
& && with \;\; G_i = \min[T, (UT_i - Up_{i0}) U_{i0}] && \\
& \sum_{j=t}^{t+UT_i-1} U_{ij} \geq UT_i \Delta_{it}^{+} && \forall i, t = G_i + 1, \ldots, T - UT_i + 1 && (12)\\
& \sum_{j=t}^{T} (U_{ij} - \Delta_{it}^{+}) \geq 0 && \forall i, t = T - UT_i + 2, \ldots, T && (13)\\
& \sum_{t=1}^{F_i} U_{it} = 0 && \forall i && (14)\\
& && with \;\; F_i = \min[T, (DT_i - Down_{i0})(1 - U_{i0})] && \\
& \sum_{j=t}^{t+DT_i-1} U_{ij} \geq DT_i \Delta_{it}^{-} && \forall i, t = F_i + 1, \ldots, T - DT_i + 1 && (15)\\
& \sum_{j=t}^{T} (1 - U_{ij} - \Delta_{it}^{-}) \geq 0 && \forall i, t = T - DT_i + 2, \ldots, T && (16)\\
& \Delta_{it}^{+} - \Delta_{it}^{-} = U_{it} - U_{it-1} && \forall i, t = 1, \ldots, T && (17)\\
& \Delta_{it}^{+} + \Delta_{it}^{-} \leq 1 && \forall i, t = 1, \ldots, T - 1 && (18)
\end{aligned}
$$

All the problem's variables but the binary ones U_{it}, Δ_{it}^{+}, Δ_{it}^{-}, are nonnegative. Constraints (1) impose that the DAM offers should not exceed the maximum quantity of energy that can be produced by each generator, while conditions (2) guarantee the satisfaction of the bilateral contracts needs by means of the zero-price DAM offers. Constraints (3) and

(4) limit the quantity to offer on the AM, for the selling and the buying bids respectively. In particular, the maximum quantity of a selling bid on the AM is at most equal to the production capacity minus the quantity of energy already accepted on the DAM. For a buying bid, we assume that the quantity can be at most equal to the selling quantity accepted on the DAM. Moreover, in order to avoid buying and selling bids on the AM at the same period for the same unit, we have introduced the additional binary variables φ_{it}^{s+} and φ_{it}^{s+} and the set of constraints (5)-(7). M is an enough big positive number.

Constraints (8) set the maximum quantity that can be offered on the ASM, according to the residual available capacity. Each of the Conditions (9) defines the quantity that has to be produced by each unit for each time period and under each scenario, after knowing all the market session outcomes. This quantity must be almost equal to the minimum quantity that can be produced by each unit (contraints (10)).

Constraints (11)-(13) represent the linear espressions of minimum up-time constraints. The set of equations (11) is related to the initial status of the units. G_i is, indeed, the number of initial periods during which unit i must be online to meet the minimum up-time requirements. The set of conditions (12) is used for the periods following G_i, and it ensures the satisfaction of the minimum up-time constraint during all the possible sets of consecutive periods of size UT_i. Finally, the set of conditions (13) is needed for the last $UT_i - 1$ periods, i.e. if a unit is started up in one of these periods, it remains online during the remaining periods. Similarly, conditions (14)-(16) provide mathematical expressions for the minimum down-time limitations.

Finally, constraints (17) and (18) are necessary to model the star-up and shut-down status of the units and to avoid the simultaneous commitment and decommitment of each unit.

3.4 Objective function

The mathematical model we are proposing aims at defining a bidding strategy for a producer that considers a tradeoff between two opposite criteria, the profits maximization and the risk minimization. For this reason, we have considered a risk-reward objective function, which is a modelling choice that is widely used in all the applicative contexts that are characterized by a high level of uncertainty. In our case, the objective function consists in a weighted sum of the expected value of the overall profits and the Conditional Value at Risk on the loss function (that will be explained later):

$$\max \sum_{v \in V} \theta^v Profit^v - \kappa CVaR_\epsilon \tag{19}$$

where κ is a user-defined trade-off parameter accounting for the risk aversion attitude, and ϵ represents the confidence level at which the Value at Risk and the Conditional Value at Risk are evaluated (usually 95% or 99%).

The overall profits for the entire planning horizon are defined as the difference between revenues and costs. The Revenues depend on the clearing prices and the quantities of energy actually cleared and, thus, are not known in advance. In particular, for each scenario v the total revenues, R^v, are the sum of the revenues from each market session (that we denote by R_{DAM}, R_{AM}, R_{ASM}, respectively) and those deriving from bilateral contracts (R_{bil}, which are constant), and can be expressed as follows :

$$R^v = R_{bil} + R_{DAM}^{p(p(v))} + R_{AM}^{p(v)} + R_{ASM}^v \qquad \forall v \tag{20}$$

$$R_{DAM}^s = \sum_{i=1}^{I}\sum_{t=1}^{T} \lambda_{it}^s \gamma_{it}^s x_{it} \qquad \forall s \tag{21}$$

$$R_{AM}^l = \sum_{i=1}^{I}\sum_{t=1}^{T} \mu_{it}^l \delta_{it}^{l+} y_{it}^{p(l)+} \qquad \forall l \tag{22}$$

$$R_{ASM}^v = \sum_{i=1}^{I}\sum_{t=1}^{T} \varsigma_t^v \rho_{it}^v z_{it}^{p(v)} \qquad \forall v \tag{23}$$

On the other side, the overall costs C^v are the sum of production costs, the cost for buying energy in the AM (C_{Prod} and C_{AM} respectively, both depend on the evolution of market outcomes) and the start-up C_{SU} and shut-down C_{SD} costs :

$$C^v = C_{Prod}^v + C_{SU} + C_{SD} + C_{AM}^{p(v)} \qquad \forall v \tag{24}$$

$$C_{Prod}^v = \sum_{i=1}^{I}\sum_{t=1}^{T} \alpha_i U_{it} + \beta_i Q_{it}^v \qquad \forall v \tag{25}$$

$$C_{SU} = \sum_{i=1}^{I}\sum_{t=1}^{T} SU_i \Delta_{it}^+ \tag{26}$$

$$C_{SD} = \sum_{i=1}^{I}\sum_{t=1}^{T} SD_i \Delta_{it}^- \tag{27}$$

$$C_{AM}^l = \sum_{i=1}^{I}\sum_{t=1}^{T} \nu_t^l \delta_{it}^{l-} y_{it}^{p(l)-} \qquad \forall l \tag{28}$$

We have considered linear functions for production costs (as argued in [17]) and constant cost coefficients for start-up and shut-down costs. The cost for buying energy on the AM has an expression similar to that of the AM revenues.

A risk management approach has been considered in our mathematical model by means of an effective and computationally efficient risk measure like the Conditional Value at Risk (or Expected Shortfall). Recently, the attention to risk management in the electricity field has grown and many interesting contributions in the literature can be found. In [18], after providing the state of the art of the risk assessment in power-system literature, the authors present Value at Risk (VaR) and heading instruments for managing market risk for suppliers, distributors and traders. A risk assessment on local demand forecast uncertainty is carried out in [19], and a daily VaR analysis is performed on a local electricity supplier using historical imbalance-settlement data in the New Electricity Trading Arrangements (NETA) system. In [20] a risk measurement approach based on VaR and CVaR has been applied to two electricity-market scenarios. In [21] a risk modelling approach based on the stochastic dominance criteria has been defined for an operation and investment planning in a power generation system.

In this work we have considered the CVaR since this risk measure has recently gained a wide consideration among practitioners in many application areas (specially in financial field) because overcomes many of the major limits of the most popular Value at Risk (see [22] and [23] for a detailed discussion on the adoption of CVaR). In particular, it allows to have a more accurate measure of potential losses and is much more tractable from a computational standpoint.

We have defined a very simple loss function representing the opposite of the profit function. Given a certain confidence level ϵ the CVaR is defined as the expected value of losses exceeding the Var:

$$CVaR_\epsilon = VaR_\epsilon + \frac{1}{1-\epsilon} E\{max[L^v - VaR, 0]\}. \tag{29}$$

Since the uncertainty within the model is represented by means of finite set of scenarios, the previous definition can be linearized, using a set of auxiliary variables and constraints, as following:

$$CVaR_\epsilon = VaR_\epsilon + \frac{1}{1-\epsilon} \sum_{v \in V} \theta^v \sigma^v \tag{30}$$

together with the following sets of constraints:

$$\sigma^v \geq L^v - VaR \qquad \forall v \tag{31}$$

$$\sigma^v \geq 0 \qquad \forall v \tag{32}$$

The overall model is a Mixed-Integer Multistage Stochastic Programming problem, with linear constraints and objective function. It is important to outline that real-life instances of this problem could have very large dimensions, due in particular to the need for a sufficiently large number of scenarios for an accurate representation of the uncertainty. For this reason, it is important to adopt an efficient solution algorithm that exploits the model peculiarity and takes into account the constraints structure.

4 Computational experiments

In order to validate the effectiveness of the proposed model, a set of preliminary computational experiments have been carried out. We have defined a simple test problem to show and comment the results. The starting basis of our experience is represented by a small GENCO that operates in the Italian market, with 3 thermo-electrical generators, whose operational characteristics are reported in Table 1.

The coefficients of the production cost function as well as the start-up and shut-down costs are the same for the three generators and assume the values $\alpha_i = 892$, $\beta_i = 14, 805$ and 43, respectively.

We have considered a time horizon of one day devided into intervals of one hour each. The uncertainty has been modelled by means of a scenario tree with a 1-10-10-3 structure,

Table 1: Technological characteristics of the generation units

	Generator 1	Generator 2	Generator 3
$Q_z^{\min}[MWh]$	0	0	0
$Q_z^{\max}[MWh]$	500	400	280
$UT_i[h]$	4	4	4
$DT_i[h]$	4	4	4
$Up_{i0}[h]$	0	0	0
$Down_{i0}[h]$	4	4	4
Location zone	North	Middle North	Sardinia

that is the root node has 10 sons, corresponding to ten possible DAM outcomes, each node at stage 1 has ten sons related to ten possible AM outcomes, and so on. The random variables have been modelled according to an analysis of historical values in the Italian market[5]. In particular, we have observed and analyzed market clearing and zonal prices of working days of January 2005. Starting from stage 1, that is the DAM session, for each hour and for each zone we have calculated the maximum and minimum values and have divided this range into ten intervals. Each interval is represented by its mean value and has a different probability of occurrence, estimated on the basis of historical observation. Moreover, we have simulated also the percentage of bid acceptance for each zonal price, associating a higher value to a lower zonal price. The combination of simulated zonal prices and bid acceptance percentage constitute the scenarios for the DAM session.

A similar procedure has been implemented for the AM, generating ten sons for each DAM scenario. However, the possibility to submit even buying bids on AM imposes to simulate not only the zonal prices but also the buying clearing price and different values of bid acceptance percentages for the different kinds of bids (buy or sell). As for the ASM, we have generated three sons for each AM scenario, simulating a unique clearing price for all the zones and three possible values of bid acceptance percentage.

The overall scenario tree has 300 leaves, corresponding each to a particular evolution of the uncertain market outcomes. We observe that the definition of a more sophisticated scenario generation technique will be necessary for big realword application but it is beyond the scope of this paper and may be the subject of future research. Interested readers are referred to [24] for a good reference on this topic.

It is also worthwhile noting the lack of general purpose solution packages for mixed-integer multistage stochastic programs. Such topic of research is indeed still in its infancy and only a limited number of algorithms have been proposed in the literature so far most of which are oriented to a specific application. A non exhaustive list includes the approaches proposed by Nowak and Römisch applied to solve the unit-commitment problem [10], by Lulli and Sen that solve the Lot sizing problem [25], by Alonso-Ayuso et al. to solve the sequencing and scheduling problem [26], and by Ahmed and Sahinidis that deals with the

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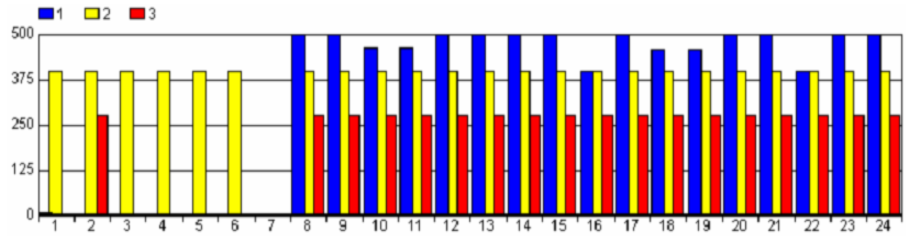

Figure 3: Units hourly bids on the DAM [MWh]

multi-stage capacity expansion planning [27]. For this reason we have implemented and solved our model by using AIMMS ([28]) as modelling environment and ILOG CPLEX ([29]) as optimization solver. The main aim is to analyze the effects of the risk aversion attitude of the GENCO on the multi-auction bidding strategy decisional process. For this purpose, a set of computational experiments have been conducted considering different values of the risk aversion parameter κ and the confidence level ϵ.

Figures 3 depicts the three units hourly bids on the Day Ahead Market. The minimum, mean and maximum zonal clearing prices on the DAM for the three zones of location of the units are reported in figure 4. It can be underlined that when clearing price is low (off peak hours 1,..,7), in general, it is preferred not to bid with all the units and to devote the capacity for other sessions. Note also that, even though the three units are similar and differ only in their maximum capacity, bids on the DAM are different, this fact is due to the different location zone.

Tables 2 and 3 report the values of expected profits and risk measure for different values of the risk aversion parameter κ and the confidence level ϵ.

Moreover, Table 4 reports the minimum and maximum value of the profits for different values of the risk aversion parameter and a fixed value for the confidence level. The reported results show how a conservative approach reduces the risk due to the market operations, but can imply a loss of profitability. More in particular, from the analysis of table 4 it is evident that, while the minimum scenario profit is the same for different values of the risk aversion parameter, the maximum scenario value is strongly sensitive to κ. In other words, the model tries to define the optimal bidding and production strategy that hedges against all the possible scenarios.

This fact could allow to a GENCO to define each time a different value of the risk aversion parameter according to the medium-term evolution of the market outcomes or to other strategic considerations.

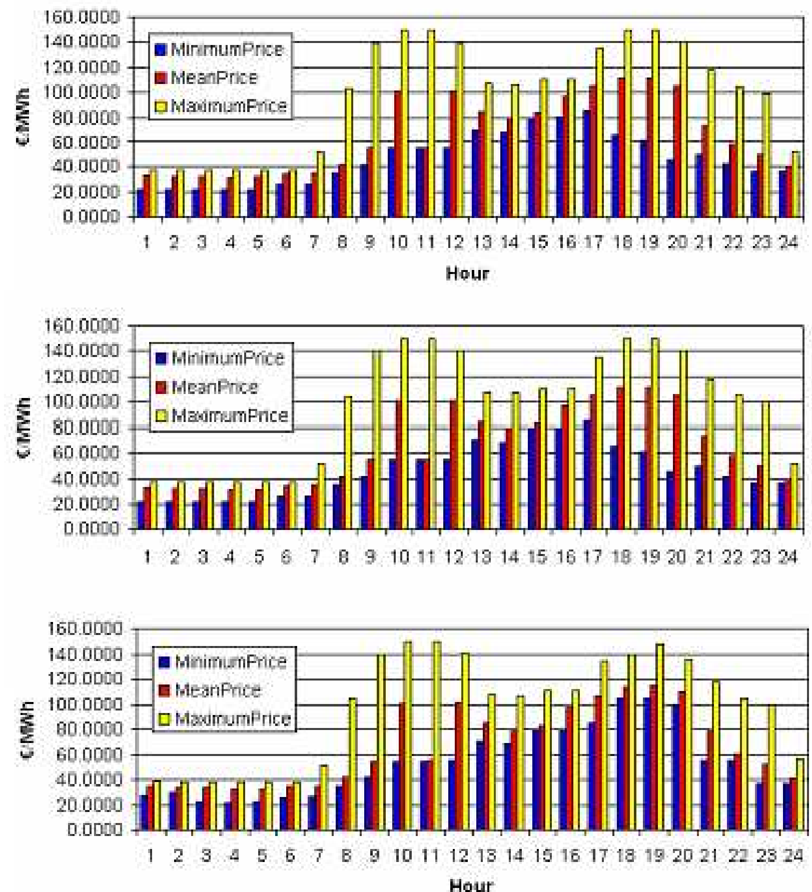

Figure 4: Minimum, mean and maximum DAM clearing price in the the zones of North, Middle-North and Sardinia [Euro/MWh]

Table 2: Results for confidence level $\beta = 0.95$

κ	E[Profit] [Euro]	CVaR [Euro]
1	1469983	57367.68
5	1434221	47101.84
10	1361978	37815.8
20	1227773	29700

Table 3: Results for confidence level $\beta = 0.99$

κ	E[Profit] [Euro]	CVaR [Euro]
1	1374260.42	59886.5
5	1336346.21	45766.27
10	1264161.3	35836.44
20	1136005.88	27417.2

Table 4: Minimum and maximum profit for confidence level $\beta = 0.99$

κ	Minimum Profit	Maximum Profit
1	337571	2445308
5	337571	2312915
10	337571	2102096
20	337571	1829522

5 Conclusion

In this paper we present and discuss a multistage stochastic formulation for the bidding problem of a GENCO operating in a liberalized electricity market. The resulting model is an integrated tool that supports the GENCO in defining the plants schedule, the bilateral contracts satisfaction and the bidding strategies while monitoring the risk related to the random market outcomes. The necessity of defining a unit-commitment based approach has introduced into the multistage model binary variables that, together with the scenario tree representation of the uncertainty, has increased the complexity of the mathematical formulation. In this paper we have focused on the validation of our decisional approach by considering a small realistic test problem that has been solved with general purpose solvers. However, for big GENCOs it will be necessary to develop a sophisticated scenario generation procedure and a specialized solution algorithm. These issues could be some of the directions for future investigations.

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