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Integrated Shipment Dispatching and Packing Problems: a Case Study

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Abstract In this paper we examine a consolidation and dispatching problem motivated by a multinational chemical company which has to decide routinely the best way of delivering a set of orders to its customers over a multi-day planning horizon. Every day the decision to be made includes order consolidation, vehicle dispatching as well as load packing into the vehicles. We develop a heuristic based on a cutting plane framework, in which a simplified Integer Linear Program (ILP) is solved to optimality. Since the ILP solution may correspond to a infeasible loading plan, a feasibility check is performed through a tailored heuristic for a three-dimensional bin packing problem with side constraints. If this test fails, a cut able to remove the infeasible solution is generated and added to the simplified ILP. Then the procedure is iterated. Computational results show that our procedure allows achieving remarkable cost savings.

Mathematics Subject Classifications (2000) 90B06 · 90B90.

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1 Introduction

In this paper we describe a case study related to the development of an optimization-based decision support system for a multinational chemical company which has to solve, on a daily basis, an integrated logistics planning problem.

The company operates a manufacturing site in Brindisi (Italy) where *oriented polypropylene* (OPP) films are produced. The final product is a film which is utilized for packaging food and other goods (toothpaste, medicines, etc). An OPP film may be metallized in order to provide special protection against oxidation, odor loss and uptake of off-odors, as it is customary, e.g., for products containing chocolate, sugar and cream. Annually more than 230,000 tonnes of OPP films are produced by the company in seven plants around the world. In Europe, films which need to be metallized are shipped from the Brindisi plant to three third-party plants, located in Termoli, Roncello and Cuneo (Italy). Finally the finished products are sent to the European distribution warehouses, located in Athus, Zeebrugge and Herstal (Belgium), from which they are shipped to the customers located in Central Europe, to Great Britain and to other European and extra-European destinations.

In this paper, we consider the decision problem associated to the transportation of the semi-finished films to third-party plants where they are metallized. Each customer order has to be shipped from Brindisi to a specific plant, depending on the required manufacturing process. Each shipment must take place within a multi-day time window, spanning from the manufacturing date (which depends on the production schedule in Brindisi) to a given deadline. Films are usually packaged into containers which may be transported by rented trucks or by common carriers. Every day, the company has to decide not only the mode of transportation but also how to consolidate its orders and when to deliver. Moreover, the manufacturer has to take care of load packing into the vehicles whenever rented trucks are chosen.

The paper is organized as follows. In Section 2, we survey the literature on integrated logistics models, with a special emphasis on transportation decisions. Then, in Section 3 we cast our shipment consolidation and dispatching problem as an optimization model. In Section 4 we propose a solution method and present some computational results.

2 Literature Review

Freight transportation plays a fundamental role along the entire logistics chain, from row material procurement up to finished good distribution. As a rule, a shipper can choose among three alternatives to transport its materials. Firstly, the company may operate a private fleet of owned or rented vehicles. Secondly, a carrier may be in charge of transporting materials through direct shipments regulated by a contract. Thirdly, the company can resort to a carrier that uses common resources (vehicles, crews, terminals) to fulfil several client transportation needs. In addition, a shipper has to design its network of transportation services at a tactical/operational level (see, e.g., [1, 8, 14, 16]). Given a set of terminals, the service network design problem amounts to deciding the features (frequency, number of intermediate stops, etc.) of the routes to be operated, the traffic assignment along these routes, the operating rules at each terminal, and possibly the relocation of empty vehicles and containers.

The objective may be the minimization of a generalized cost taking into account a combination of carrier's operating costs and customers' expectations. In particular, the service network design problem involves freight consolidation.

Consolidation can be achieved in three ways. Firstly, small shipments that have to be transported over long distances may be consolidated so as to transport large shipments over long distances and small shipments over short distances (*facility consolidation*). Secondly, less-than-truckload pick-up and deliveries associated with different locations may be served by the same vehicle on a multi-stop route (*multi-stop consolidation*). Thirdly, shipment schedules may be adjusted forward or backward so as to make a single large shipment rather than several small ones (*temporal consolidation*). In our study the second and third types of consolidation affect the planning process. Moreover, planning a transportation service at an operational level may involve bin packing issues (see, e.g., [12]). Depending on the characteristics of the products and on the transportation mode, single items or boxes have to be mounted onto a pallet or inserted in a container, pallets have to be loaded onto trucks, or containers have to be put on a ship or on a plane.

While the literature on freight transportation planning is well-established (see, e.g., [5] and [6] for surveys), at present there exist just few contributions which consider several issues at the same time in an integrated fashion. This is the case of Iori et al. [10], which have recently embedded multi-dimensional packing features in a classical capacitated VRP, and of Bertazzi et al. [2–4], which have studied integrated inventory and distribution problems. Also see [15] for a review of integrated analysis of production-distribution systems.

In this paper we present a multi-day integrated model for a freight transportation problem including order consolidation and dispatching as well as load packing. We also develop a heuristic based on a cutting plane framework, in which a simplified Integer Linear Program (ILP), neglecting bin packing constraints, is solved to optimality. Since the ILP solution may correspond to an infeasible loading plan, a feasibility check is performed through a tailored heuristic for a three-dimensional bin packing problem with side constraints. If this test fails, a cut able to remove the infeasible solution is generated and added to the simplified ILP. Then the procedure is iterated. In order to keep the ILP size moderate, a rolling horizon technique is used.

3 Problem Formulation

In this section, we present a mathematical formulation of our problem. A producer has to choose the best way to deliver timely a set of K orders to its set N of customers over a planning horizon of T days. The decisions to be taken are:

- Which is the best mode of transportation for each shipment;
- How orders have to be consolidated;
- Which is the schedule for each rented/owned truck (start times, intermediate stops (if any), the order in which stops are visited, etc.);
- How to load each of the rented/woned truck.

Each order $k \in K$ is characterized by a destination $i_k \in N$, a release time d_k (the day in which order k is ready for delivery), and a deadline D_k (the day within which order k must be delivered to i_k). Moreover, each order k is composed of a set C_k of containers each of them (say, $j \in C_k$) having a weight $q_{jk} \geq 0$, a width w_{jk}, a height h_{jk} and a depth l_{jk}. Thus, the total weight q_k and volume v_k of an order k are, respectively, given by:

$$q_k = \sum_{j=1}^{|C_k|} q_{jk}; \quad v_k = \sum_{j=1}^{|C_k|} w_{jk} h_{jk} l_{jk}$$

The company has two alternatives for shipping the orders: it may rent 'one-way' truck trips, or use a Less Than Load (LTL) carrier. A rented truck may follow any route r of a preestablished set R. To each route $r \in R$ are associated a set of stops $N_r \subseteq N$ (visited in a given order), a fixed cost f_r, and a number of traveling days τ_{kr} to deliver order k. Moreover, by assuming that to each route r is assigned a priori a vehicle, we can also characterize a route by its maximum capacity Q_r (maximum weight to be carried by the vehicle operating route r), and its volume V_r given as the product of its width W_r, its height H_r and its depth L_r. The second shipment alternative, i.e. LTL carrier, is simply characterized by a variable cost g_k that depends on (the weight and the volume of) order k and by the number of days τ'_k that order k takes to reach its destination. The problem's decision variables are:

- y_{rt}: An integer variable equal to the number of trucks operating route r on day t;
- x_{krt}: A binary variable equal to 1 if order k is assigned to route r starting on day t, and 0 otherwise;
- z_k: A binary variable equal to 1 if order k is transported by a common carrier, and 0 otherwise.

It is clear that, for each $k \in K$, variable z_k is defined only if $d_k + \tau'_k \leq D_k$. Hence, our problem can be represented mathematically by the following model:

$$Minimize \quad \sum_{r \in R} \sum_{t=1}^{T} f_r y_{rt} + \sum_{k \in K} g_k z_k \tag{1}$$

$$\sum_{k: d_k \leq t \leq D_k - \tau_{kr}, i_k \in N_r} q_k x_{krt} \leq Q_r y_{rt}, r \in R, t = 1, \ldots, T \tag{2}$$

$$\sum_{r: i_k \in N_r} \sum_{t: d_k \leq t \leq D_k - \tau_{kr}} x_{krt} + z_k = 1, k \in K \tag{3}$$

$$x_{krt} \in \{0, 1\}, k \in K, r \in R, t = 1, \ldots, T \tag{4}$$

$$y_{rt} \geq 0, \text{ integer}, r \in R, t = 1, \ldots, T \tag{5}$$

$$z_k \in \{0, 1\}, k \in K \tag{6}$$

$$z_k = 0, k \in K, d_k + \tau'_k > D_k \tag{7}$$

$$\text{'packing'constraints} \tag{8}$$

The objective function (1) is the total cost paid to transport the orders through either a rented vehicle or a common carrier. Constraints (2) state that the total weight carried on route r must not exceed capacity Q_r times the number of trucks operating route r on day t. Constraints (3) impose that each order should be assigned to a route operated by a rented truck or to a common carrier, but not both. Finally packing constraints (8) may be very complex and therefore are here considered implicitly. It is easy to show that, when packing constraints are removed, formulation (1)–(8) can be transformed into a network design model on a time-expanded network.

4 Solution Method

The problem presented in the previous section is well-known to be NP-hard. The presence of the packing constraints makes its solution particularly difficult with exact methods. For this reason, our approach consists in applying a tailored cutting plane approach. First, a relaxation of models (1)–(8) including some surrogate packing constraints is solved. If the resulting solution is infeasible for the original problem then the current solution is cut off by adding a new constraint, and the new relaxation is solved again. This procedure is then repeated until a feasible solution is found.

More specifically, the packing constraints are first substituted by the following set of relaxed constraints:

$$\sum_{k:d_k \leq t \leq D_k - \tau_{kr}, i_k \in N_r} v_k x_{krt} \leq V_r y_{rt}, r \in R, t = 1, \ldots, T \tag{9}$$

Constraints (9) impose that the total volume of the orders to be transported along route r on day t should not exceed the volume of the vehicles operating that route. These are, indeed, a relaxation of the packing constraints in which the characteristics (size, orientation, etc.) of both items and trucks are not taken into account. Solving (1)–(7), (9) determines a lower bound on the original objective function (1) and provides a solution

$$(\overline{x}_{krt}, \overline{y}_{rt}, \overline{z}_k)$$

that may violate the packing constraints. Indeed, even if constraints (9) are satisfied, solution $(\overline{x}_{krt}, \overline{y}_{rt}, \overline{z}_k)$ may not correspond to a feasible loading plan. For this reason, it is necessary to implement, at each step of the solution method, a procedure that checks whether the solution satisfies the original constraints.

4.1 Feasibility Checking Procedure

The feasibility checking procedure can be summarized as follows:

<u>For</u> $r \in R$

  <u>For</u> $t = 1, \ldots, T$

    Solve a packing problem with items associated to orders $\{k \in K : \overline{x}_{krt} = 1\}$
    <u>If</u> the number of required vehicles is greater than $\overline{y}_{rt}$ <u>then</u> the solution $(\overline{x}_{krt}, \overline{y}_{rt}, \overline{z}_k)$ is declared infeasible.

An infeasible solution $(\overline{x}_{krt}, \overline{y}_{rt}, \overline{z}_k)$ can be cut off by means of the following constraint:

$$\sum_{k \in K} \overline{x}_{krt} x_{krt} \leq \sum_{k \in K} \overline{x}_{krt} - 1, r \in R, t = 1, \ldots, T \tag{10}$$

4.2 Constructive Heuristic for the Packing Problem

In order to verify whether the current solution is feasible or not a packing problem has to be solved for each $r \in R$ and for each $t = 1, \ldots, T$. It is worth noting that if each packing problem was solved exactly the overall algorithm would be exact. In this paper we approach the packing problems heuristically. Indeed we make use of TSpack, the unified Tabu Search code proposed by Lodi et al. [13], in which we embed a tailored constructive heuristic. In what follows we refer to trucks as bins and to orders as items. The main features of our packing problems are:

- Items and bins are considered to be two-dimensional since all items and bins have the same height;
- Item rotation is allowed;
- Items need to be loaded into the bins in such a way that each of them can be unloaded at destination without unloading items to be delivered at subsequent stops.

The tailored constructive heuristic is as follows:

Step 1: Enumerate all the feasible combinations of items in the truck's width direction (taking into account that each item can be rotated);
Step 2: Sort the combinations according to an utilization rate;
Step 3: Sort the items $\{k \in K : \overline{x}_{krt} = 1\}$ according to the sequence of destinations along route r;
Step 4: Scan, for each destination, the list of combinations and create as few shelves as possible; these shelves are feasibly allocated to trucks as they are needed.

4.3 Rolling Horizon Technique

Problem (1)–(8) is a integer linear program, whose size depends on $|K|$ and $|R|$ as well as on the planning horizon duration T. In practice, even for relatively small values of these parameters, problem (1)–(8) may he hard to solve. To overcome this difficulty, we adopted a rolling horizon technique in which problem (1)–(8) is solved over a reduced planning horizon made up of T' $(< T)$ days and the set of orders $\bar{K}' \subset \bar{K}$ is formed by the 'most urgent' orders. Given T', orders $\bar{K}'$ are determined by suitably ranking orders according to their release dates and deadlines.

Obviously the solution of the reduced problem is particularly meaningful at the beginning of the reduced planning horizon while it is less reliable at the end of the period. For this reason the problem is solved each day by shifting the planning horizon one day ahead.

Table I The set of daily routes and the corresponding cost (in Euro)

	Route r	f_r
1	Brindisi–Termoli	400.00
2	Brindisi–Roncello	750.00
3	Brindisi–Cuneo	800.00
4	Brindisi–Termoli–Roncello	780.00
5	Brindisi–Termoli–Cuneo	830.00
6	Brindisi–Roncello–Cuneo	830.00
7	Brindisi–Termoli–Roncello–Cuneo	860.00

5 Computational Results

We evaluated the performance of our approach by carrying out a dynamic simulation, spanning a six-month planning horizon. First of all, we performed a preliminary analysis in order to set parameters T and $|\bar{K}'|$. It turned out that, for $T \leq 5$ and $|\bar{K}'| \leq 10$, subproblems were easier to solve but the heuristic solutions compared unfavorably with the hand-made solutions. On the other hand, greater values of T and $|\bar{K}'|$ made subproblems very hard to solve. As a result, we set $T = 7$ and $|\bar{K}'| = 15$. R was made up of the routes covering one or more destinations (third-party metallizers). Among the routes that included more than one metalizer only those corresponding to a shortest Hamiltonian path were considered. In Table I we list the daily routes together with the corresponding fixed costs f_r. In Table II we

Table II Computational results for a sample month

Day	Cuts	LB/UB	LB time	UB time
3	9	0.90	32	90
4	0	1.00	10	10
5	2	0.95	3	5
6	0	1.00	5	5
7	1	0.93	7	10
10	0	1.00	20	20
11	2	0.98	7	10
12	0	1.00	2	2
13	0	1.00	2	2
14	0	1.00	2	2
17	0	1.00	2	2
18	1	0.98	3	5
19	0	1.00	2	2
20	2	0.97	3	5
21	0	1.00	2	2
24	0	1.00	70	70
25	0	1.00	40	40
26	0	1.00	2	2
27	0	1.00	2	2
28	0	1.00	5	5

Table III Heuristic versus hand-made solutions

Month	Heuristic solution cost (Euro)	Hand-made solution cost (Euro)
1	14,210.00	15,672.00
2	16,750.00	18,017.00
3	22,840.00	23,717.00
4	24,620.00	26,644.00
5	21,180.00	22,614.00
6	25,340.00	26,842.00

describe the computational experiment for a sample month. The column headings are as follows:

- Cuts: Number of cutting constraints;
- LB/UB: The ratio between the lower bound (provided by problem (1)–(7), (9)) and the upper bound provided by the heuristic;
- LB time: The computing time (in seconds) of the lower bounding procedure;
- UB time: The computing time (in seconds) of the heuristic.

These results show that the ratio between the lower and upper bounds is greater than 0.98 on the average and that the computing time never exceeds 2 min. Of course, we expect that the computing time grows fast as the number of destination increases. We leave the study of these aspects as a future research topic. Moreover, in Table III we compare, for each month, our solution with the hand-made solution. These results show that our approach outperforms the procedure actually used by the company. Greater savings are expected from our procedure if applied to all users (both metalizers and end-user).

6 Conclusions

In this paper we have addressed a multi-day consolidation and dispatching problem in which vehicle packing constraints are considered explicitly. Our algorithm is based on a cutting plane framework that first relaxes the packing constraints and then checks their satisfaction by means of a tailored heuristic. The method has been applied to solve a real-world problem. Results show that our algorithm allows achieving significant savings with respect to hand-made procedures. We expect that the approach presented in this paper may result in impractical computing times as the number of destinations increases. As a result, an interesting research problem to be addressed in the future is to devise efficient metaheuristics for large scale problems. A further research topic is related to the integration of the distribution and manufacturing decision-making procedures. Indeed job scheduling at the manufacturing plant determines order release dates at the distribution stage. Thus a coordination between the two phases can be beneficial in order to reduce transportation cost.

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