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Constrained auction clearing in the Italian electricity market

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Abstract. Most of the liberalized electricity systems use the auction as a market model. The complexity of the underlying optimization formulation depends on the technical and regulatory constraints that must be considered. In Italy, the auction clearing should include not only congestion management limitations, but also a challenging regulatory constraint imposing that, while the zonal prices are allowed on the selling side, a uniform purchasing price has to be applied for all the zones of the Italian system. Such constraint introduces several complexities such as non-linearity and integrality. In this paper we discuss the modeling issues arising in the Italian context and we propose, in addition, a mechanism for the priority management of the offers/bids acceptance. We test the behavior of the models developed on a set of problems that represent all the possible scenarios that can be met in practice. The numerical results demonstrate the validity and the effectiveness of the proposed models.

Key words: Electricity market, auction clearing, optimization models

AMS classification: 90-20, 90C90

1 Introduction

The recent advent of deregulation in the electricity market has introduced many new challenges, particularly on the management side. In the restructured markets, power generation, trading and distribution are ensured on competitive basis rather than being a privilege of a monopolistic utility. In this direction, even with some delay with respect to other European Union partners, Italy has started a deep restructuration

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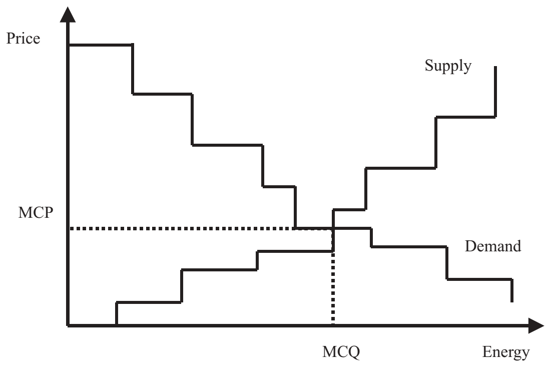

Fig. 1. Market clearing information

process towards the liberalization of its electricity business. The so-called *Decreto Bersani*, a law issued in 1999 as an application to the European Directive CE 96/92, represents a normative and operative framework for the progressive liberalization of the Italian market. The delay in the deregulation has allowed the Italian legislator to be informed not only about the experience of the already restructured markets worldwide but also about the recent advances achieved in the field of electricity markets design (such as Cameron and Cramton 1999; Rassenti et al. 2001; and Cramton 2003). A key role in the new structure is played by an independent grid operator, known as GRTN (*Gestore di Rete di Trasmissione Nazionale*), whose main purpose is to provide a non-discriminatory open access to the grid while ensuring the security and the reliability of the system. Another important operator is the GME (*Gestore del Mercato Elettrico*) whose role is to manage the short-term forward electricity exchanges by using the auction as a market model for the price definition.

More specifically, the GME collects the bids/offers submitted by the different players and provides the market clearing information. Every bid and offer is characterized by a pair price–quantity. The market clearing price (MCP) and quantity (MCQ) for each hour of the next day are determined at the intersection of the aggregated supply and demand curves (Fig. 1). Mathematically, the auction equilibrium can be represented as an optimization problem having the objective of maximizing the total energy quantities exchanged while ensuring the strict matching between the energy bought and sold. The resulting model, called unconstrained model, is linear and generally of trivial solution. Examples of application of this model can be found in the clearing of the spot market in the Scandinavian system, in the clearing of the daily auction organized by the Spanish market operator, and also in the day ahead Californian market before its failure (a detailed description of the such markets could be found in ENEA 2000).

Other market structures, however, may require the inclusion of additional technical and regulatory constraints that may increase the complexity of the model.

Technical constraints may involve the congestion management by means of nodal flow limitations. This happens when one or more lines of the transmission network are unable to accommodate the energy to be exchanged between the network's nodes. A possible solution, thus, is to define nodal prices, as applied in the PJM system (Mansur 2001), rather than a unique equilibrium price for the whole network. Alternatively, many systems implement the zonal pricing model, as an approximation of the full nodal pricing, by aggregating the nodes into zones (Bjorndal et al. 2002). Zonal pricing model is applied, for example, in the Australian market (Read and Cattopadhyay 1999), in the internal Norwegian system (Bjorndal and Jornsten 2001) and also in the Italian system at the end of a three years transitorily period (GME 2002a). Meanwhile, the equilibrium model in Italy should include a challenging regulatory constraint imposing that, while the zonal prices are allowed on the selling side, a uniform purchasing price has to be applied for all the zones of the Italian system. Such constraint introduces inevitably several complexities such as non-linearity of the model and integrality of the decision variables. Moreover, in presence of this restriction, the integration of a mechanism for the priority management of the marginal offers/bids acceptance becomes a crucial issue introducing remarkable difficulties.

This paper deals with the mathematical formulation of the complex auction equilibrium problem generated in the Italian context. The literature related to equilibrium models for the worldwide electricity markets seems to be very limited mainly because of confidentiality reasons (Read 2003). Read and Chattopadhyay (1999) describe the zonal pricing model applied in the Australian market but without giving any formulation for it. A simple form of zonal model is found in Bjorndal and Jornsten (2001) as a description to the pricing mechanism in the internal Norwegian market. Finally, the last work we are aware of is a GME report proposing an equilibrium model for the Italian market during its transitorily period (GME 2002b). The present paper can be considered as an extension and a substantial improvement of the model presented by the GME. We describe first our basic model which is a slight modification of the GME formulation, and then we discuss its main shortcomings to motivate the improving features leading to our complete approach.

The paper is organized as follows. In the next section we introduce a basic version of the market equilibrium model with uniform purchasing price and zonal selling prices and then we report some improvements. In Sect. 3 we propose an approach for the priority management of the marginal offers/bids acceptance. This feature, even though very important for transparency matters, has been ignored in the GME report. Section 4 will be devoted to the experimental results and then some remarks will conclude the paper.

2 Auction equilibrium model for the Italian system

In this section, we describe the design of the optimization model proposed to support the decision making process defined by the selling offers and purchasing bids acceptance procedure.

First of all, we give a detailed description of our notation. Then, starting from the fundamental requirements of the process, we formulate a basic version of the model describing the structure and analyzing the main properties. Moreover, by taking into account some further requirements, we are able to give a new and improved version of the model, which achieves the maximization of the clearing quantity.

2.1 Notation

For the mathematical representation of the model, we adopt the notation detailed in the sequel. As we have already mentioned, the Italian power transmission system is organized by geographical zones (currently there are six zones); each zone is denoted by the index k, with $k = 1, \dots K$. The different zones are interconnected by H connections, each of them denoted by the index h, with $h = 1, \dots H$. An interzonal connection h can represent either a single transmission line (as the case of the two major islands with the rest of the country) or a path of several transmission lines between two zones. We denote by indices i and j respectively the purchasing bids and the selling offers, where, for each zone k, $i = 1, \dots I_k$ and $j = 1, \dots J_k$. Moreover, we introduce the following parameters:

- $\underline{PQP}_i^k$, $\overline{PQP}_i^k$: minimum and maximum quantity of energy associated to bid i, within zone k;
- $\underline{SQP}_j^k$, $\overline{SQP}_j^k$: minimum and maximum quantity of energy associated to offer j, within zone k;
- PP_i^k, SP_j^k: purchasing and selling prices associated with bid i and offer j, within zone k;
- $\underline{PF}_h$, $\overline{PF}_h$: minimum and maximum power flow on connection h;
- CPF_h^k: contribution of zone k to the power flow on connection h. The values of this parameter depend on the topology of the network. In the Italian case, which has a tree connection network, they are limited to be either 0 or 1.

Finally, the decision variables are the following:

- pb_i^k, sb_j^k: binary variables, which assume the value 1 if, respectively, the purchasing bid i and the selling offer j, within zone k, are accepted, and zero otherwise;
- PQP_i^k, SQP_j^k: quantity of energy accepted from bid i and offer j, within zone k;
- SPZ^k: selling price for zone k;
- PPZ: purchasing price for all the zones (national purchasing price).

Within the general inter–zonal flow management framework we remark that we need, basically, to satisfy the following general requirements:

– the total social surplus over all the zones must be maximized;
– the acceptance decision must be based on some priority ordering criterium of the offers/bids, in order to guarantee a correct and equitable behavior of the electricity market;
– the definition of a uniform purchasing price, over all the zones, that has to be as low as possible, in order to guarantee the efficiency of the market.

2.2 Basic model formulation

We start by describing a basic version of the optimization model. The total social surplus can be defined as the following quantity:

$$\sum_{k=1}^{K}\left(\sum_{i=1}^{I_k} PP_i^k\ PQP_i^k - \sum_{j=1}^{J_k} SP_j^k\ SQP_j^k\right),$$

and, in order to meet the first of the above requirements, this quantity have to be maximized.

The following constraints impose the range of values that the quantity of energy associated to the accepted offers/bids can achieve:

$$\underline{PQP}_i^k\ pb_i^k \le PQP_i^k \le \overline{PQP}_i^k\ pb_i^k,\ i = 1,\dots I_k,\ k = 1,\dots K, \tag{1}$$

$$\underline{SQP}_j^k\ sb_j^k \le SQP_j^k \le \overline{SQP}_j^k\ sb_j^k,\ j = 1,\dots J_k,\ k = 1,\dots K, \tag{2}$$

A technical condition, imposing upper and lower bounds on the power flow between any two zones, must be stated as follows:

$$\underline{PF}_h \le \sum_{k=1}^{K} CPF_h^k \left(\sum_{j=1}^{J_k} SQP_j^k - \sum_{i=1}^{I_k} PQP_i^k\right) \le \overline{PF}_h,\ h = 1,\dots H. \tag{3}$$

The next constraints set the condition that the selling price for zone k must be at least equal to the maximum selling price associated to the accepted offer j within zone k.

$$SPZ^k - SP_j^k\ sb_j^k \ge 0,\ j = 1,\dots J_k,\ k = 1,\dots K. \tag{4}$$

Moreover, we impose that the selling price for the zone k must be less or equal to the price of any accepted purchasing bid, by means of the constraints:

$$PP_i^t - SPZ^k pb_i^t \ge 0,\ i = 1,\dots I_t,\ k = 1,\dots K,\ t = 1,\dots K. \tag{5}$$

The following constraint defines the energy balance between purchasing bids and selling offers:

$$\sum_{k=1}^{K}\sum_{i=1}^{I_k} PQP_i^k - \sum_{k=1}^{K}\sum_{j=1}^{J_k} SQP_j^k = 0, \tag{6}$$

and we match also the cash flow balance between the purchasing and selling revenues by:

$$PPZ\sum_{k=1}^{K}\sum_{i=1}^{I_k} PQP_i^k - \sum_{k=1}^{K} SPZ^k \sum_{j=1}^{J_k} SQP_j^k = 0. \tag{7}$$

Finally, the constraints

$$pb_i^k \geq pb_l^k,\ k = 1, \dots K,\ i = 1, \dots I_k,\ l = i+1, \dots I_k, \tag{8}$$

$$sb_j^k \geq sb_m^k,\ k = 1, \dots K,\ j = 1, \dots J_k,\ m = j+1, \dots J_k, \tag{9}$$

set the priority among the bids and the offers of the same zone, respectively. We suppose here that, within each zone, the bids (offers) are ordered so that the priority decreases as $i\ (j)$ increases.

2.3 Model improvements

On the basis of some preliminary analysis and validation of the proposed basic model, it has been evident that, in some real situations, the behavior of the model could be not satisfactory. For instances, let us consider the case in which the supply and demand curves overlap each other for an horizontal segment: in this specific case, the most suitable result might be the acceptance of all the offers/bids on the overlapping horizontal segment. This result is not ensured by the output of the basic model.

In order to impose this kind of behavior, we propose to introduce some perturbation to the supply curve in such a way to remove the overlapping segment with the demand curve and get a single crossing point. Moreover, the crossing point must be shifted to the right as far as possible, in order to further increase the total social surplus and, hence, maximize the total number of accepted offers/bids.

This can be modeled by adding to the previous objective function the following quantity:

$$\sum_{k=1}^{K}\sum_{j=1}^{J_k} \alpha\ SQP_j^k,$$

and by tuning the parameter α (by assigning to it a small real value with comparisons to the difference between the prices associated to two consecutive offers), it is possible to get rid of the overlapping segment and shift the market clearing quantity to include all the feasible offers/bids.

As pointed out in the general requirements previously stated, another suitable behavior of the model is to keep as low as possible the national purchasing price. This objective is not ensured by the basic model but can be achieved by simply adding the term $-PPZ$ to the objective function. This settling forces constraints (4) to be active at the optimum, since minimizing PPZ implies the minimization of SPZ^k (from constraint (7)) and this implies (from constraints (4)) that $SPZ^k = max\{SP_j^k\}$, for the accepted selling offers.

Furthermore, by considering new constraints given by the following conditions:

$$PQP_i^k \geq pb_l^k\ \overline{PQP}_i^k,\ k = 1,\dots K,\ i = 1,\dots I_k,\ l = i+1,\dots I_k,$$
$$SQP_j^k \geq sb_m^k\ \overline{SQP}_j^k,\ k = 1,\dots K,\ j = 1,\dots J_k,\ m = j+1,\dots J_k,$$

we meet the requirement imposing that a given offer (bid) of any zone can be accepted only if the higher priority offers (bids) of the same zone have been already totally accepted.

To summarize, the complete formulation of the improved version of the model is the following:

$$\max \left(\sum_{k=1}^{K} \sum_{i=1}^{I_k} PP_i^k\ PQP_i^k - \sum_{k=1}^{K} \sum_{j=1}^{J_k} SP_j^k\ SQP_j^k + \sum_{k=1}^{K} \sum_{j=1}^{J_k} \alpha\ SQP_j^k - PPZ \right)$$

$s.t.$

$$\underline{PQP}_i^k\ pb_i^k \leq PQP_i^k \leq \overline{PQP}_i^k\ pb_i^k,\ i = 1,\dots I_k,\ k = 1,\dots K \tag{10}$$

$$\underline{SQP}_j^k\ sb_j^k \leq SQP_j^k \leq \overline{SQP}_j^k\ sb_j^k,\ j = 1,\dots J_k,\ k = 1,\dots K \tag{11}$$

$$\underline{PF}_h \leq \sum_{k=1}^{K} CPF_h^k \left(\sum_{j=1}^{J_k} SQP_j^k - \sum_{i=1}^{I_k} PQP_i^k \right) \leq \overline{PF}_h,\ h = 1,\dots H \tag{12}$$

$$SPZ^k - SP_j^k\ sb_j^k \geq 0,\ j = 1,\dots J_k,\ k = 1,\dots K \tag{13}$$

$$PP_i^t - SPZ^k\ pb_i^t \geq 0,\ i = 1,\dots I_t,\ k = 1,\dots K,\ t = 1,\dots K \tag{14}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{I_k} PQP_i^k - \sum_{k=1}^{K} \sum_{j=1}^{J_k} SQP_j^k = 0 \tag{15}$$

$$PPZ \sum_{k=1}^{K} \sum_{i=1}^{I_k} PQP_i^k - \sum_{k=1}^{K} SPZ^k \sum_{j=1}^{J_k} SQP_j^k = 0 \tag{16}$$

$$PQP_i^k \geq\ pb_l^k\ \overline{PQP}_i^k,\ k = 1,\dots K,\ i = 1,\dots I_k,\ l = i+1,\dots I_k \tag{17}$$

$$SQP_j^k \geq\ sb_m^k\ \overline{SQP}_j^k,\ k = 1,\dots K,\ j = 1,\dots J_k,\ m = j+1,\dots J_k \tag{18}$$

$$pb_i^k = \{0,1\},\ i = 1,\dots I_k,\ k = 1,\dots K \tag{19}$$

$$sb_j^k = \{0, 1\},\ j = 1, \dots J_k,\ k = 1, \dots K \tag{20}$$

$$PQP_i^k \geq 0,\ i = 1, \dots I_k,\ k = 1, \dots K \tag{21}$$

$$SQP_j^k \geq 0,\ j = 1, \dots J_k,\ k = 1, \dots K \tag{22}$$

$$SPZ^k \geq 0,\ k = 1, \dots K \tag{23}$$

$$PPZ \geq 0. \tag{24}$$

The proposed model lies in the framework of mixed integer nonlinear programming model, even though the structure of nonlinearities (only in (15) and (17)) is quite simple. The model can be characterized by large scale dimensions (depending on the number of offers/bids to manage) and requires, typically, a fast solution since the time horizon of an auction clearing is quite short.

3 Priority acceptance modeling

It is worth noting that, in order to fully guarantee the fairness and stability of the electricity market, we have to require an unique optimal global solution provided by the proposed auction equilibrium model. In fact, it is quite easy to recognize that, in some cases, the model may find multiple optimal solutions, since it does not impose the respect of any priority criterion over all the zones (as it is required by the Italian current regulation). For instance, this specific situation can occur when near the intersection point there are purchasing bids, or selling offers, having the same price, that have not been accepted or have been partially accepted, violating a global priority criteria over all the zones (we denote these offers/bids as *critical*, see Fig. 2). In this situation, we have to reallocate the energy quantities in order to impose the respect of a priority criterion over all the zones and, hence, to obtain a well defined and unique optimal solution. To this end, we propose a *second stage* model formulation that, taking in input suitable information coming from the solution of the auction equilibrium model (*first stage* model), rebalances, in a feasible way, the energy quantity associated to the *critical* offers/bids, in order to satisfy a global priority criterion over all the zones. The *second stage* model can be instantiated, for both the cases purchasing bids and selling offers, by considering slightly different formulations. Some attention should be paid, in the case of the selling offers, to the fact that the reallocation of the energy quantity among the offers with the same price can affect the value of the selling price for some zones and, hence, the national purchasing price.

Before proceeding, we give some further details on the notation. Since the sets of *critical* offers/bids are subsets of the total sets, respectively, of purchasing bids $\{1, \dots I_k\}$ and selling offers $\{1, \dots J_k\}$, we denote the *critical* offers/bids by the set of indexes $\{l_k, \dots u_k\}$, whose meaning will be clear by the context. Moreover, we indicate with a hat the values of the quantities which represent the output of the *first stage* model and that are assumed to be input data for the *second stage* model.

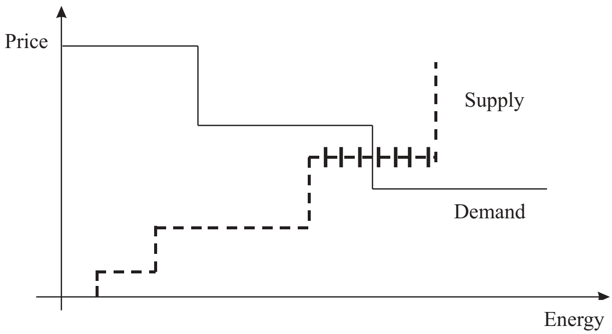

Fig. 2. Critical offers/bids

The formulation of the model in the case of the purchasing bids is the following:

$$\max \sum_{k=1}^{K}\sum_{i=l_k}^{u_k} c_i^k \; PQP_i^k \tag{25}$$

$s.t.$

$$\underline{PQP}_i^k \; pb_i^k \le PQP_i^k \le \overline{PQP}_i^k \; pb_i^k, \; i = l_k, \ldots u_k, \; k = 1, \ldots K, \tag{26}$$

$$\underline{PF}_h \le \sum_{k=1}^{K} CPF_h^k \left(\sum_{j=1}^{J_k} \widehat{SQP}_j^k - \sum_{\substack{i=1 \\ i \ne [l_k,\ldots u_k]}}^{I_k} \widehat{PQP}_i^k - \sum_{i=l_k}^{u_k} PQP_i^k \right) \le \overline{PF}_h, \quad h = 1, \ldots H, \tag{27}$$

$$\sum_{i=l_k}^{u_k} PQP_i^k - \sum_{i=l_k}^{u_k} \widehat{PQP}_i^k = 0, \; k = 1, \ldots K, \tag{28}$$

$$pb_i^k = \{0, 1\}, \; i = l_k, \ldots u_k, \; k = 1, \ldots K, \tag{29}$$

$$PQP_i^k \ge 0, \; i = l_k, \ldots u_k, \; k = 1, \ldots K. \tag{30}$$

The formulation of the model related to the selling offers is the following:

$$\max \sum_{k=1}^{K}\sum_{j=l_k}^{u_k} c_j^k \; SQP_j^k \tag{31}$$

$s.t.$

$$\underline{SQP}_j^k \; sb_j^k \le SQP_j^k \le \overline{SQP}_j^k \; sb_j^k, \; j = l_k, \ldots u_k, \; k = 1, \ldots K, \tag{32}$$

$$\underline{PF}_h \le \sum_{k=1}^{K} CPF_h^k \left(\sum_{\substack{j=1 \\ j \ne [l_k,\ldots u_k]}}^{J_k} \widehat{SQP}_j^k + \sum_{j=l_k}^{u_k} SQP_j^k - \sum_{i=1}^{I_k} \widehat{PQP}_i^k \right) \le \overline{PF}_h,$$

$$h = 1, \dots H, \quad (33)$$

$$SPZ^k - SP_j^k\ sb_j^k \geq 0,\ j = 1, \dots J_k,\ k = 1, \dots K, \quad (34)$$

$$PP_i^t - SPZ^k\ pb_i^t \geq 0,\ i = 1, \dots I_t,\ k = 1, \dots K,\ t = 1, \dots K, \quad (35)$$

$$PPZ \sum_{k=1}^{K} \sum_{i=1}^{I_k} PQP_i^k - \sum_{k=1}^{K} SPZ^k \Big(\sum_{\substack{j=1 \\ j \neq [l_k, \dots u_k]}}^{J_k} \widehat{SQP}_j^k + \sum_{j=l_k}^{u_k} SQP_j^k \Big) = 0, \quad (36)$$

$$\sum_{j=l_k}^{u_k} SQP_j^k - \sum_{j=l_k}^{u_k} \widehat{SQP}_j^k = 0,\ k = 1, \dots K, \quad (37)$$

$$sb_j^k = \{0, 1\},\ j = l_k, \dots u_k,\ k = 1, \dots K, \quad (38)$$

$$SQP_j^k \geq 0,\ j = l_k, \dots u_k,\ k = 1, \dots K, \quad (39)$$

$$SPZ^k \geq 0,\ k = 1, \dots K, \quad (40)$$

$$PPZ \geq 0. \quad (41)$$

In both the above models, the weights c_i^k and c_j^k, defined in such a way $c_i^k > c_{i+1}^k$ and $c_j^k > c_{j+1}^k$, aim to reallocate the accepted quantity of energy imposing the priority among the relevant offers/bids. This trivially follows from the structure of the objective functions, which have to be maximized in both models. The precise setting of the weights has to be choosen by the end user, according, in general, to the order of magnitude of the relevant quantities in the models (typical choice could be defined by setting a sequence monotonically decreasing of one order of magnitude or less).

As far as the constraints are concerned, we observe that, besides the obvious conditions on the feasible range of values that the quantity of energy associated to the accepted offers/bids can achieve and on the power flow between the zones, the quantity of energy to reallocate among the *critical* offers/bids must be equal to the relevant quantity of energy determined by the *first stage* model.

As a final remark, we note that, in order to istantiate the *second stage* models we have to know the *critical* offers/bids and, in particular, the relevant set of indexes $l_k, \dots u_k$. This can be easily done by using some off–line procedure that we do not describe in this paper.

4 Computational experiments

Since the quite original nature of the proposed models, the computational experiments have been planned and carried out with the basic aim to assess the suitability, reliability and semantic behavior of the same models. To this end, we have considered several test problems designed to face all possible situations that can occur in practice, with respect to the intersection of the supply and demand curves. Before

Table 1. Maximum flow values (in MWh) between any two zones

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Zone 1	0	4800	0	0	0	0
Zone 2	4800	0	300	2500	0	0
Zone 3	0	300	0	0	0	0
Zone 4	0	2500	0	0	2200	0
Zone 5	0	0	0	2200	0	1000
Zone 6	0	0	0	0	1000	0

introducing the test problems, we observe that each offer/bid submitted to the energy market should present the following features: the type of operation it refers to (i.e. purchasing bid or selling offer), the price, the maximum acceptable quantity, and the geographical zone. Furthermore, the GME should define a priority order based on price, and for equal price values we assume that the priority order is based on the time of offer/bid's submission, even though other criteria may be adopted. In our experiments, we have assumed that the offers/bids are already ordered (as the result of a pre-processing phase) and thus each offer/bid is labeled according to its priority order and the geographical zone it belongs to.

We have considered six different test problems, differing in the way the intersection between supply and demand curves happens. The defined test problems represent all the possible scenarios that the GME can meet in practice. The characteristics of the offers and bids –prices and quantities– and the partition of the national territory in geographical zones have been simulated by considering the data reported in GME (2001a), and GME (2001b). For the sake of simplicity, we have not included the virtual zones, which represent the exchange with the foreign systems interconnected to the Italian network. Thus, we have assumed that the Italian territory is divided into sixth zones and the maximum flow value between any two zones is reported in Table 1.

In the definition of the test problems we have made the following assumptions which, while realistic, do not cause loss of generality:

– for each offer/bid the minimum acceptable quantity is fixed to 10% of its maximum amount;
– because of the bi-directionality of the energy flow, the minimum value between any two zones has been fixed to the opposite of its maximum value.

All the computational experiments have been carried out by using the system software LINGO, version 6.0, on a Windows based personal computer.

The presentation of the numerical results is not a trivial task. Indeed, reporting the objective function value, or the values of the decision variables does not immediately convince about the validity of the proposed models. In order to clarify the presentation of the results, we proceed as follows: first we present the results obtained by solving the model without flow constraints. By observing the curve representing the specific test problem and analyzing the results, it is possible to

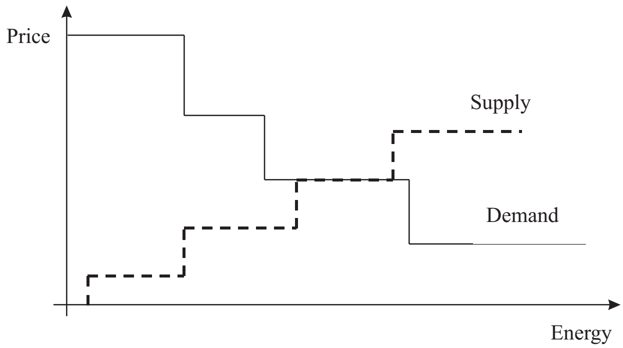

Fig. 3. Test problem 1

verify the feasibility and optimality of the determined solution. Then, we re-solve the model by adding the flow constraints. Their effect is to limit the amount of energy accepted to a value less than 100% because of the flow restriction between zones. For the sake of brevity, we report the complete results for a test problem only: interested readers are referred to Beraldi et al. (2003) for the complete collection of results for all the test problems.

The selected test problem depicts a situation that can occur very often in practice and fully explains the behavior of our model. This test problem is represented by the supply/demand curve depicted in Fig. 3. The two curves intersect along an horizontal line. This specific structure suggests that all the selling offers until the last contact point between the curves ("limit" point for short) should be accepted. Furthermore, energy should be assigned to the purchasing bids until the "limit" point, respecting the priority order. To this end, the application of the "second stage" model to the purchasing bids is required. Table 2 and Table 4 report the numerical results. In particular, for each bid or offer we report the geographical zone (Zone), the zonal identification number (ZIN), the global identification number (GIN) defined on the basis of the total priority order, the unit price (UP) (in Euro/MWh), and the percentage of energy accepted in both the first and second stages of the method (% 1st and % 2nd). Note that the second stage of the method is applied to the purchasing bids only. Table 4 reports the zonal selling prices, the national purchasing price and the objective function value.

It is worthwhile noting that the effect of the application of the second phase of the method is the redistribution of the energy previously assigned to the critical bids. Indeed, part of the energy assigned to bid 23 by the first stage of the method is assigned to bid 17. This confirms our initial assumptions: an offer/bid may be accepted only if all the offers/bids with greater priority are fully accepted, except for flow congestion problems.

Let now consider the model with flow constraints (all the results are identical to those of Table 2 except for the bids reported in Table 4). The major effect produced by the inclusion of these restrictions refers to bid 13, now accepted at the percentage

Table 2. Unconstrained results for test problem 1

Purchasing bids						Selling offers				
Zone	ZIN	GIN	UP	% 1st	% 2nd	Zone	ZIN	GIN	UP	% 1st
1	1	1	80	100		1	1	1	50	100
1	2	5	75	100		1	2	6	55	100
1	3	11	70	100		1	3	12	60	100
1	4	17	65	83.3	100	1	4	13	60	100
1	5	24	60			1	5	20	65	100
1	6	30	55			1	6	23	70	
2	1	6	75	100		2	1	2	50	100
2	2	12	70	100		2	2	7	55	100
2	3	18	65	100	100	2	3	14	60	100
2	4	19	65	100	100	2	4	21	65	100
2	5	25	60			2	5	28	75	
2	6	31	55			2	6	31	80	
3	1	2	80	100		3	1	3	50	100
3	2	7	75	100		3	2	8	55	100
3	3	13	70	100		3	3	22	65	100
3	4	26	60			3	4	24	70	
3	5	35	50			3	5	32	80	
4	1	3	80	100		4	1	4	50	100
4	2	8	75	100		4	2	9	55	100
4	3	14	70	100		4	3	15	60	100
4	4	20	65	100		4	4	25	70	
4	5	27	60			4	5	29	75	
4	6	32	55			5	1	5	50	100
5	1	9	75	100		5	2	10	55	100
5	2	15	70	100		5	3	16	60	100
5	3	21	65	100	100	5	4	17	60	100
5	4	22	65	100	100	5	5	26	70	
5	5	28	60			5	6	30	75	
5	6	33	55			6	1	11	55	100
6	1	4	80	100		6	2	18	60	100
6	2	10	75	100		6	3	19	65	100
6	3	16	70	100		6	4	27	70	
6	4	23	65	100	50					
6	5	29	60							
6	6	34	55							

Table 3. Changes introduced by the inclusion of the flow constraints in test problem 1

Purchasing bids					
Zone	ZIN	GIN	UP	% 1st	% 2nd
3	3	13	70	87.5	
6	4	23	65	100	75

of 87.5%. Furthermore, for bid 23 the percentage of energy accepted (by the second stage of the method) is now 75 instead of 50 of the unconstrained case. As expected the introduction of the flow constraints causes a worsening of the objective function value that takes now the value of 199646.

Table 4. Numerical results for test problem 1

SPZ^1	SPZ^2	SPZ^3	SPZ^4	SPZ^5	SPZ^6	PPZ	Objective value
65	65	65	60	60	60	63, 56	200146

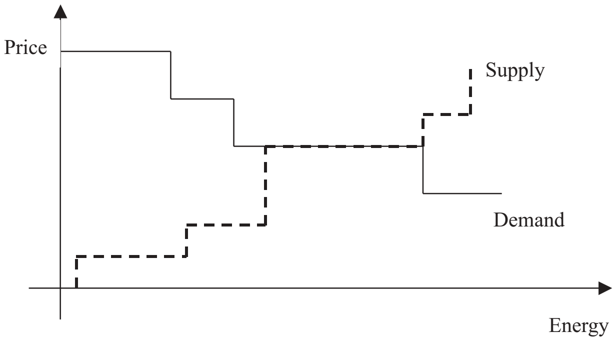

Fig. 4. Test problem 2

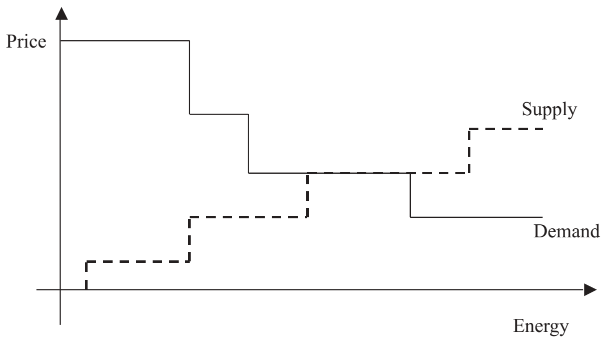

Fig. 5. Test problem 3

Let now discuss the results of the other test problems. The second test problem is similar to the first one (Fig. 4). The configuration suggests to accept all the purchasing bids and selling offers until the "limit" point. However, we observe that, because of the flow constraints, some offers/bids on the horizontal line may not be fully accepted, even though they could be suitable.

In the third test problem (Fig. 5) the supply and demand curves have a common horizontal line. The solution suggests to accept all the purchasing bids and selling offers until the "limit" point according to the priority order. We observe that, the application of the second phase of the method for the selling offers is required in order to determine the zonal selling prices (SPZ^k) and the national purchasing price (PPZ).

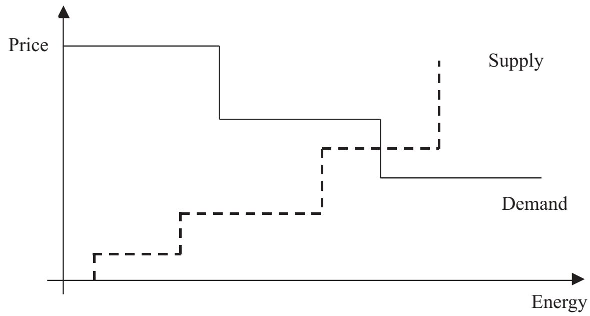

Fig. 6. Test problem 4

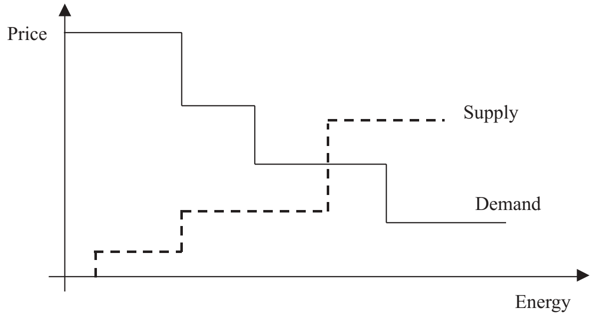

Fig. 7. Test problem 5

The fourth test problem looks different from the previous ones (Fig. 6). Here the two curves in one point of an horizontal segment of the supply curve and a vertical segment of the demand curve. In this case energy is assigned to the purchasing bids until the intersection point and to the selling offers according to the priority and to flow constraints. Analogously to the third test problem the application of the second phase of the method for the selling offers is necessary.

The fifth test problem is the reciprocal of the previous one (Fig. 7). The system's configuration requires to accept all selling offers until the intersection point and to assign the energy to purchasing bids with respect to the priority order and flow constraints. This intuition is confirmed by the numerical results (see Beraldi et al. 2003).

The last test problem is depicted in Fig. 8. Here the cumulative curves have a vertical segment in common. Actually, even though this situation seems to be quite unusual, it has been included for the sake of completeness. The solution suggests the satisfaction of all the offers/bids, until the energy level exchanged on the common vertical segment. We observe that because of the flow restrictions some offers/bids are not fully accepted.

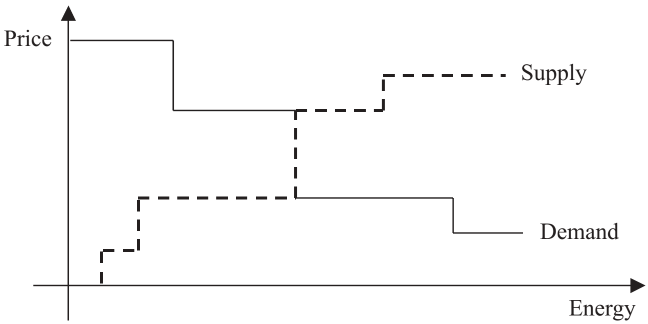

Fig. 8. Test problem 6

5 Concluding remarks

The recent restructuration of the Italian electrical power system imposes a quite unusual condition on the market clearing: while sellers are paid on zonal basis, buyers should pay the same price in all the zones. This regulatory requirement implies many difficulties in the development of suitable optimization models aimed to control and manage the auction clearing.

Under this respect, the main goal of this paper is to propose effective modeling solutions which can face the several issues and scenarios arising in the Italian electricity market operation, in order to achieve economic efficiency while respecting fairness and transparency requirements.

The proposed models have been tested on a set of self generated real–base problems (which represent all the possible events occurring during market operations), by using a well–known software system able to handle non–linearity and integrality. The numerical results demonstrate, for all the instances of the test problems considered, the validity of the proposed models, both in terms of correctness and effectiveness of the obtained optimal solutions.

As final remark, we observe that we did not emphasize, in this paper, some very important issues related to the computational side of the proposed models. In fact, even though the structure of the proposed models can be considered quite simple, real-world instances could be quite difficult to solve due to their large scale dimensions in terms of number of offers/bids to be managed. That could required the development of well suited algorithms and computational techniques, which be also able to deal with the need of real-time solution. Under this respect, high performance computing could be a very useful tool. All this will be the subject of a future work as soon as the number of transitions in the Italian market will motivate it.

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