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Optimal capacity allocation in multi-auction electricity markets under uncertainty

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Abstract

The advent of competitive markets confronts each producer with the problem of optimally allocating his energy/capacity so as to maximize his profits. The multiplicity of auctions in electricity markets and the non-trivial constraints imposed by technical and bidding rules make the problem of crucial importance and difficult to model and solve. Further difficulties are represented by the dynamic and stochastic natures that characterize the decision process. We formulate the problem as a multi-stage mixed-integer stochastic optimization model under the assumption that the seller is a price taker. We validate the effectiveness of the proposed model on a representative test problem.

Keywords: Interrelated electricity markets; Bidding strategies; Price-taking seller; Unit commitment; Multi-stage stochastic programming

1. Introduction

It is widely believed that opening electricity markets for competition is the preferred way to reduce the costs and improve the service quality. In this direction, several countries around the world have proposed various restructuring processes aimed at liberalizing their electricity business. In addition, there are countries that are still in the phase of defining the normative and operative framework for the progressive liberalization of their electricity markets. Different market models have been adopted with the aims of attaining economic efficiency and optimizing social welfare. In the model we are considering in this paper, a key role is played by the independent grid operator (IGO) whose

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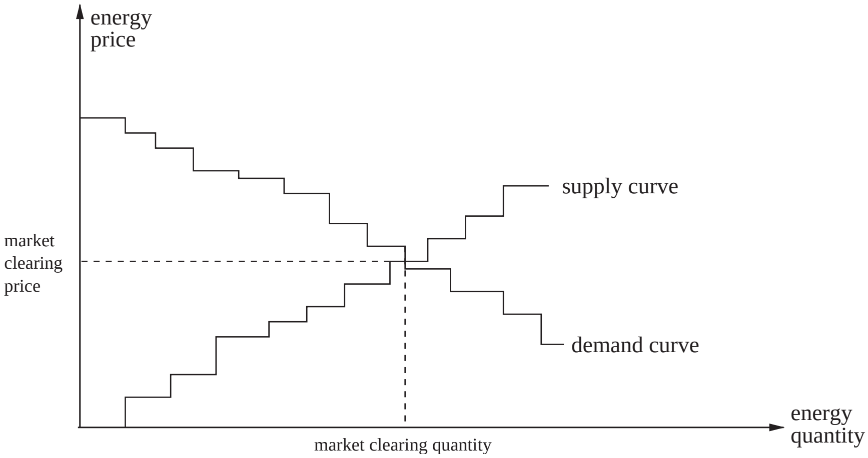

Fig. 1. Market clearing price definition.

principal goal is to provide non-discriminatory open access transmission services while ensuring the security and the reliability of the system. The IGO carries out his job in close collaboration with the electricity market operator (EMO) whose role is to manage the short-term forward electricity markets that make use of auction mechanisms. [1] Specifically, the EMO collects the offers and bids submitted by the various players and determines the market clearing equilibrium for each market and each time period. The offers and bids are used to construct the aggregated market supply and demand curves, respectively. Fig. 1 shows examples of these curves. The EMO is responsible for disseminating the market clearing information to all market participants and interested parties.

The EMO may operate markets of different forward horizons. In this paper, we focus on the day-ahead and successive markets with each operating day divided into hourly periods. For each period, there are separate auctions for energy and the different types of reserves/capacity-based ancillary services. All the auctions are run independently and their clearing prices, for the same operating period, may differ considerably. Consequently, the decisions on the quantities to offer in each market become a critical issue in the desire of each seller of maximizing his profits [1].

A supplier must determine in which markets and in what quantities to offer his energy and/or capacity. We refer to this as the capacity allocation problem. This problem has two salient characteristics. It is dynamic such that decisions are time period dependent. The problem is also stochastic such that the most important data—the clearing prices and quantities for each period—are not known in advance. Mathematical programming under uncertainty represents an effective tool to solve a problem with these characteristics.

Power production under uncertainty has been the subject of intensive research for the solution of short-, mid- and long-term planning problems. Most of the literature deals with the well-known unit commitment or capacity expansion problems for the vertically integrated utility structure that preceded liberalization [2–4]. The restructuring has created the necessity for revised models and for

[1] In certain jurisdictions, the IGO assumes the EMO responsibilities; one such example is the PJM interconnection (www.pjm.com).

the development of appropriate solution procedures. Indeed, new formulations for the unit commitment problem have been proposed for the different market models adopted [5,6]. The nature of the assumptions used, such as deterministic vs. stochastic and oligopoly vs. a perfectly competitive market, leads to the main differences among the proposed formulations. Unit commitment-based models have also been formulated to determine bidding strategies that support the seller in the definition of his supply function [7–10]. In this context, most of the papers consider a single electricity market environment and, at most, include the consideration of long–term contracts but ignore the sequential opportunities for offering the generators' output for the different markets that may exist. Little attention has been paid to the definition of capacity allocation models for supply offers in multi-auction and/or multi-market environments. In Marmiroli et al. [11] have solved the generation scheduling problem in a deterministic environment under multiple electricity markets. The resulting capacity allocation model is a quadratic programming problem with explicit representation of the status of each generation unit.

In this paper, we construct an integrated framework within which we can explicitly consider the multiple interrelated markets for electricity. We present an optimization model that defines a multi-auction capacity allocation strategy which is optimal with the explicit representation of uncertainty. Specifically, we include the on–off unit decisions, technical/physical limitations and market constraints to formulate a multi-stage stochastic model that supports a seller in determining the quantities to allocate to each auction so as to maximize his profits. This work solves a very real and important problem; it constitutes an effective application—possibly the first—of stochastic programming techniques to the capacity allocation problem for the restructured electricity markets. A salient characteristic of the proposed approach is the choice of the decision moments of the multi-stage model. Unlike in previous works which use decision stages to correspond to the quantities to offer to the same market over the time periods of the decision horizon, each stage in our model corresponds to the quantity offered to each auction for the same operating period. We refer to this choice as an auction-wise formulation to differentiate it from the conventional period-wise multi-stage models. The market clearing information of each auction represents the uncertain data that, once observed, will constitute the basis for the decisions to adopt in the next auction. All the decision stages refer to the same operating period, and the inter-temporal constraints in the model arising from the minimum up/down restrictions of each unit are explicitly represented.

We formulate the problem under the fundamental assumption that the seller is a price taker and has no possibility/intention to exercise market power to affect the auctions' prices. This assumption is realistic for markets with many small sellers and without collusion and barriers to entry. As a natural consequence, each seller has incentives to offer the units' output at its marginal costs [12] excluding any pure financial interest behind his participation. Therefore, the problem considered here does not concern the definition of a supply function characterized by complex price-quantity combinations. Rather, it addresses the way of formulating the offers of a seller to each of the auctions in order to maximize profits.

The paper is organized as follows. The next section is devoted to the description of the structure and the main features of the electricity market that we consider as a setting for the problem. We then introduce the scenario tree formulation and we present a mathematical model for the stochastic capacity allocation problem in Section 3. Section 4 is dedicated to the experimental results to illustrate the model's performance. We conclude with some remarks and discuss future work.

2. Market structure overview

We provide a description of the basic market structure in which the sellers submit their offers. This structure is rather generic and can be extended to include specific characteristics of actual implementations in various venues. The market structure here incorporates the essential characteristics of the recently restructured Italian market [13]. The description serves the aim of the work, which is to formulate the capacity allocation strategy of a price-taking seller who submits offers in multiple competitive electricity markets. [2]

The seller can participate in the following auctions:

- *Day-ahead energy market* (*DEM*): Buyers and sellers participate by submitting supply offers and demand bids. A separate supply offer is submitted for each production unit with eventually multiple block components and associated prices forming a non-decreasing function.
- *Adjustment market* (*AM*): This is an energy market that allows both sellers and buyers to adjust their day-ahead schedules on the basis of new information about load forecast and unit status. This market consists of multiple sessions. In certain jurisdictions, such a market is run every hour and so the AM is, in effect, the hour-ahead market.
- *Day-ahead reserves market* (*DRM*): This is similar to the DEM auction even though here only the sellers are allowed to bid for the provision of the spinning and non-spinning reserves to meet the demand defined by the IGO. The offers for reserves are capacity–price pairs with prices expressed in $/MW/h; there is no energy supply associated with this offer. Any energy eventually dispatched will be paid at a rate established by the relevant balancing market.
- *Balancing market* (*BM*): this is a real-time market that ensures the load-supply energy balance around the clock. In this market both reduction and increase of the energy are allowed. Reduction (increase) is achieved by reducing (increasing) the production or increasing (reducing) the consumption of energy. Such adjustment are determined on the basis of the merit order of offers.

Faced with this diversity of electricity markets, a seller has to consider several aspects in order to determine the optimal capacity allocation of each unit as shown in Fig. 2. In addition to the

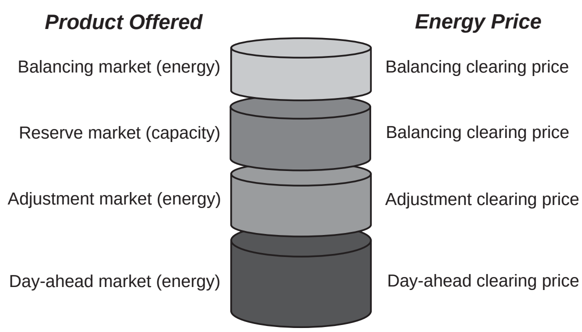

Fig. 2. Unit's capacity allocation.

[2] Even though energy and capacity can be traded as two different products, we assume that any block of power will never have (incorrectly) double payment in capacity and energy markets.

restrictions imposed by the units' physical/operational limitations and market regulations, the seller may want to include strategic objectives such as diversification and market niche. The complexity of the problem increases as the number of units increases and also as more markets are considered.

The capacity allocation problem becomes even more complex when uncertainty is explicitly incorporated into the model. Some of the most important data are inherently uncertain: at the beginning of every auction, the clearing price and quantities bought/sold by each buyer/seller are not known. An optimal decision strategy needs to take into account the whole range of possible values that these variables may assume since any formulation based on the mean value or on the worst-case analysis may lead to unacceptable approximations.

There is a range of methodologies to deal with the optimization process under uncertainty [14]. We next formulate the capacity allocation problem as a multi-stage stochastic programming model.

3. The capacity allocation model

In order to simplify the discussion of the model, we restrict our attention to only three interrelated markets as representative of the spectrum of possible markets with different horizons. As a concrete example, we consider the DEM, the AM and the BM for real time. The extension to include more markets such as the DRM and other eventual AM sessions is straightforward and simply involves additional variables with the corresponding constraints but presents no other challenges in terms of incorporating them into the model discussed here. In addition, we suppose to operate in a transmission unconstrained environment.

We denote by t, $t=1,\dots,T$, the periods of the operating day and we assume that the seller has I thermal production units. Each unit i, $i=1,\dots,I$ is characterized by its minimum capacity q_i and its maximum capacity Q_i. The seller has to decide, for each time period, which units to commit, and the quantities to offer in each of the three auctions in order to maximize his profits. The decision process is made under uncertainty since the amounts of energy effectively sold/bought depend on the clearing prices. Consequently, the seller must develop his strategy with the considerations of all the uncertain events that may occur. Fig. 3 shows the bidding process for the three markets. The scheme indicates different moments of decision phases leading to the definition of a multi-stage stochastic model.

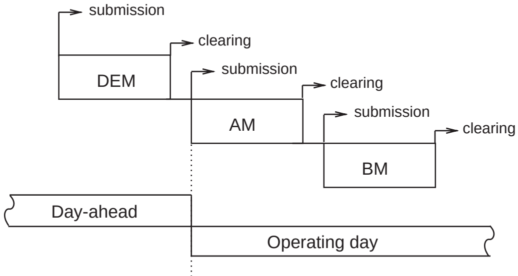

Fig. 3. Sequence of offers' submission.

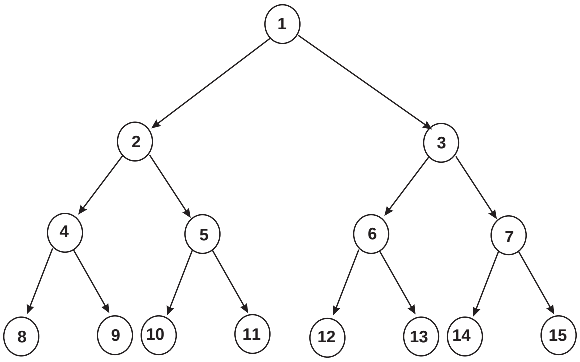

Fig. 4. Scenario tree corresponding to the first test problem.

In this paper, we will use the intuitive scenario tree formulation to represent the evolution of the random clearing prices and quantities. On the basis of our auction-wise representation, each time period will be characterized by its own scenario tree. Fig. 4 shows an example of a scenario tree with eight scenarios. The root node stands for the first stage and corresponds to the immediately observable deterministic data. The nodes in successive stages correspond to possible outcomes that capture the evolution of the random variables by which uncertainty is mathematically represented. We associate a probability value with each node to represent its likelihood of occurrence. Each node i, except the root node, has a unique immediate predecessor $p(i)$ in the preceding stage and a finite number of successors in the next stage. Nodes without any successors are called the leaves of the tree. They are in a one-to-one correspondence with the scenarios: a scenario is a path from the root node to a leaf and represents a joint outcomes of the problems' data over all the auctions (stages).

Nodes are also associated with the sequential decision process, so that to each node corresponds a decision variable that depends on the previous decisions and the scenario so far observed as well.

The proposed model, based on the scenario tree formulation, represents a variant of the multi-stage stochastic programming framework. [3]

Initially, the decision maker has to determine which units to commit and the quantities to offer in the DEM auction. More specifically, we denote by u_i^t the binary variable that takes the value 1 if unit i is committed in the period t, and 0 otherwise. If $u_i^t = 1$, x_i^t will represent the first-stage decision on the quantity of energy to offer in the DEM auction by using the unit i.

The model can also be extended to the case in which the decision maker wishes to offer different blocks of energy at different prices. In this case, it is enough to consider each block as the maximum capacity of an independent unit having a variable cost equal to the offering price. A new binary variable is introduced for each block to determine whether or not to offer that block in one of the auctions and additional constraints are added to ensure the consistency of the commitment decisions for blocks of the same unit.

First-stage constraints: The technical constraints that arise in the unit commitment problem concern both u_i^t and x_i^t. In this formulation, we include the minimum up and minimum down times and

[3] For an introductory survey on multi-stage stochastic programs the reader is referred to [14,15].

the capacity constraints. Given the minimum up time D_i and the minimum down time d_i the former constraints can be expressed, for each unit i, in terms of the binary variable u_i^t as follows:

$$(u_i^{t-1} - u_i^t)(U_i^{t-1} - D_i) \geqslant 0 \quad \forall t,$$

$$(u_i^t - u_i^{t-1})(U_i^{t-1} + d_i) \leqslant 0 \quad \forall t,$$

where the state variable U_i^t represents the cumulative information about the number of hours unit i has been on/off up to hour t. We should notice that, even though very simple and intuitive, the above constraints are non-linear. However, it is possible to substitute each of them by a set of linear constraints, as proposed in [16], at the cost of a remarkable increase of the problem's size.

Constraints related to the variables x_i^t require that the maximum capacity Q_i of unit i not be exceeded once the unit i is turned on

$$x_i^t \leqslant Q_i u_i^t \quad \forall i \ \forall t.$$

Second-stage constraints: Once the DEM clears and the effective amount of energy sold in the DEM is observed, the seller must decide the quantities to offer in the other auctions. It may be convenient not only to offer the quantities preserved since the beginning to the two auctions but also re-offer the quantity of energy remaining not sold in the DEM auction. More specifically, we observe that the quantity effectively sold in the DEM may be expressed as a percentage γ of the quantity offered in that market. Three cases can occur in dependence of the value of the DEM clearing price with respect to the offer price of each unit i:

- the offer price is less than the clearing price: in this case all the quantity of energy x_i^t offered will be sold and, thus, $\gamma_i = 1$;
- the offer of unit i determines the market clearing price and so it is a marginal unit: in this case only the fraction $\gamma_i x_i^t$ of the offered quantity is cleared in that market, with γ_i that can assume any value in the range $0 < \gamma_i \leqslant 1$;
- the offer price of unit i is greater than the clearing price and so is unsuccessful in that market: the quantity of energy sold will be zero ($\gamma_i = 0$).

The different values that can be assumed by γ_i will define the possible alternatives resulting from the DEM for unit i. By combining all the alternatives for all the units we define the set of the DEM outcomes at the period t and we denote its cardinality by S^t. We use the index s with $s = 1, \ldots, S^t$ to denote a generic node of the second stage of the scenario tree at time period t. In the sequel, we will add the subscript s and the superscript t to the term γ_i and also to the second-stage decision variables in order to identify the observed outcome s at the period t.

On the basis of the realized outcome s, the seller should decide, for each period $t = 1, \ldots, T$, the quantities to offer to the AM and BM auctions which are optimal with respect to all the possible S^t events.

These quantities, that we denote by $y_{i,s}^t$ and $z_{i,s}^t$, respectively, can be assured by, eventually, turning on uncommitted units or increasing the production of some already committed units provided that the maximum capacity constraints are satisfied

$$y_{i,s}^t + z_{i,s}^t \leqslant Q_i u_i^t - \gamma_{i,s}^t x_i^t \quad \forall s \ \forall i \ \forall t.$$

While $z^t_{i,s}$ will represent a first allocation for the BM auction that will be followed by another corrective action, the term $y^t_{i,s}$ will be, in correspondence to the outcome s, the effective quantity to offer to the AM auction. Neither term is deterministic but depends on the particular outcome s that has been observed in the DEM auction.

Third-stage constraints: Once the effective amount of energy sold in the AM is observed, the seller makes a third-stage decision in terms of its offer to the BM. Once again, for each time period t, we use the index $l=1,\ldots,L^t$ to denote an outcome observed after the clearing of the AM auction and we express the quantity effectively sold in the AM auction as a percentage $\delta^t_{i,l}$ of the quantity offered $y^t_{i,s}$. The third-stage decision variable $z^t_{i,l}$ will indicate the additional quantity of energy to offer in the BM auction under the outcome l at period t. In the BM auction, the seller has the possibility to commit unused units or increase the output of one or more units under the constraint of respecting the units' capacities

$$z^t_{i,l} \leqslant Q_i u^t_i - \gamma^t_{i,p(l)} x^t_i - \delta^t_{i,l} y^t_{i,p(l)} - z^t_{i,p(l)} \quad \forall l \ \forall i \ \forall t.$$

Here $p(l)$ denotes the predecessor of node l in the scenario tree. For each time period $t=1,\ldots,T$ and each outcome l, the sum $(z^t_{i,p(l)}+z^t_{i,l})$ will be the effective quantity to offer in the BM auction. As in previous cases, not all the quantity offered is necessarily successful in the auction, but it depends on the outcome that materializes. For each time period t, we denote by v^t, with $v^t = 1,\ldots,V^t$, a generic outcome that can be verified after the BM auction clearing, and by $\rho^t_{i,v}$ the percentage of the quantity of energy effectively sold of unit i. Since our model considers only three markets, the quantity offered but not sold in the BM will represent a loss of opportunity. The quantity $(1-\rho^t_{i,v})(z^t_{i,p(l)}+z^t_{i,l})$ could be planned to an alternative use such as offered in a neighbour market [12] or dedicated for the pumping phase in hydro pumped-storage plants. We shall consider such loss of opportunity as an additional cost to be introduced as a penalty term in the objective function.

Other constraints: In addition to the above constraints, the model may include explicit limitations on the hourly output change of each unit. This allows that the total change in the output of every unit in two consecutive periods respects its ramp rates.

Minimum capacity limits should also be respected; the total quantity produced by unit i during the period t and under all the possible scenarios of the problem should match at least the minimum capacity to be produced by unit i:

$$x^t_i + y^t_{i,p(l)} + z^t_{i,p(l)} + z^t_{i,l} \geqslant q_i u^t_i \quad \forall l \ \forall i \ \forall t.$$

Finally, it is necessary to include non-negativity limitations. Indeed, since our model does not consider financial trading and does not include the uncertainty related to the possible failures in the production units, it implicitly assumes that the seller is interested only in selling energy and will not need to buy energy from the adjustment and balancing markets. As a consequence, all the variables of the problem can assume only non-negative values.

Objective function: The objective is to maximize the total profits over the operating day. For each period, the profits are defined as the difference between revenues and costs. Revenues depend on the clearing prices and the quantities of energy effectively sold and, thus, are not known in advance. The expected revenues are instead considered. For this, we need to introduce additional notation. Let η^t_s, π^t_l and θ^t_v denote, for each period t, the probability of occurrence of scenarios s, l and v related to the three markets, respectively. Furthermore, let λ^t_s, μ^t_l and ζ^t_v denote the outcomes of the random clearing prices in the three markets during the time period t. Thus, the objective function to maximize is the sum of the expected profits deriving from the three auctions minus the costs and the loss of opportunity term

$$\max \sum_{i=1}^{I} \sum_{t=1}^{T} \left\{ -f_i^t u_i^t + \sum_{s=1}^{S^t} \eta_s^t [\lambda_s^t \gamma_{i,s}^t x_i^t - C_i(\gamma_{i,s}^t x_i^t)] + \sum_{l=1}^{L^t} \pi_l^t [\mu_l^t \delta_{i,l}^t y_{i,p(l)}^t - C_i(\delta_{i,l}^t y_{i,p(l)}^t)] \right.$$
$$\left. + \sum_{v=1}^{V^t} \theta_v^t [\zeta_v^t \rho_{i,v}^t (z_{i,p(p(v))}^t + z_{i,p(v)}^t) - C_i(\rho_{i,v}^t (z_{i,p(p(v))}^t + z_{i,p(v)}^t)) - C'^t (1 - \rho_{i,v}^t)(z_{i,p(p(v))}^t + z_{i,p(v)}^t)] \right\}.$$

Here $C_i(.)$ denotes the variable cost function of unit i, f_i^t is the fixed cost related to the decision of turning on the unit i, and C'^t is the opportunity cost at time period t. The variable cost may be expressed as a polynomial function of the unit output. In practice, this function is often approximated by a piece-wise linear function or simply truncated o the affine form (as pointed out in [5]).

3.1. Model extensions

Different additional features could be included into the basic model in order to better express the decision maker preferences and to be able to cover the different circumstances of the production reality with the appropriate formulation.

Strategic constraints: These constraints are used to model the seller's preferences in the weight of participation in the different markets. By using these kind of constraints, the seller can give indicative guidelines on how to allocate the available capacity among the different markets. It is possible, for example, to impose that the total quantity of energy to offer to the DEM during period t should be greater than a given value, say H^t. The model may include, in this case, the following constraint:

$$\sum_{i=1}^{I} x_i^t \geqslant H^t.$$

Analogously, similar constraints can be formulated for the other two auctions in terms of given values, as above, or as a function of the quantities offered to the DEM. It is possible to impose, for example, that the total quantity to offer to the AM auction should be at most a percentage α^t (with $0 \leqslant \alpha^t \leqslant 1$) of that offered to the DEM. This means that the following constraints:

$$\sum_{i=1}^{I} y_{i,s}^t \leqslant \alpha^t \sum_{i=1}^{I} x_i^t$$

should be satisfied for each time period t and scenario s.

Market constraints: Electricity markets may have some specific rules that impose some regulatory constraints on the behaviour of the operators. In the Italian market, for example, the instructions permit to a seller to participate in the AM and BM auctions only if he has sold energy in the DEM [13]. In other words, the components of the vectors $y_{p(l)}^t$ and $z_{p(l)}^t + z_l^t$ should be set to zero if the components of the vector $\gamma_{p(l)}^t x^t$ are all null. These logical relations are usually formulated by introducing additional binary variables. Alternatively, in our case we can combine these constraints with the strategic constraints and use simply the following expressions:

$$\sum_{i=1}^{I} y_{i,s}^t \leqslant \alpha_1^t \sum_{i=1}^{I} \gamma_{i,s}^t x_i^t \quad \forall t \ \forall s,$$

$$\sum_{i=1}^{I} (z_{i,p(l)}^t + z_{i,l}^t) \leqslant \alpha_2^t \sum_{i=1}^{I} \gamma_{i,p(l)}^t x_i^t \quad \forall t \ \forall l$$

with $0 \leqslant \alpha_1^t, \alpha_2^t \leqslant 1$.

Risk management: The decision maker may be also interested in hedging financial risks due, for example, to high prices volatility or generation failures. The risk management may be included into the model by dynamically managing futures and options contracts to minimize expected shortfall. A careful representation of this feature may require deep investigation. For instance some insights could be found, for example, in [17].

3.2. Solution approaches

The capacity allocation model is a stochastic multi-stage mixed-integer program (SMIP) with nonlinear constraints. To the best of our knowledge, no specific solution approaches have ever been proposed to solve this class of problems. In effect, even for the linear SMIP (by substituting the up/down constraints by their linearized counterpart) the scientific literature is rather limited and has been mainly focused on the two-stage case (see the survey proposed in [18]). Lokketangen and Woodruff [19] applied a heuristic in which the progressive hedging algorithm is combined with a tabu search heuristic. Caroe and Schultz [20] proposed a Lagrangian relaxation within a branch and bound algorithm. In addition to the methods designed to solve general linear SMIP, there exist solution strategies specifically tailored to solve the unit commitment problem. Such problem, which is closely related to the one considered in this paper, has been the subject of intensive research in the last 30 years. Among the different solution strategies, the most successful technique seems to be the Lagrangian relaxation (see for example [2,3,7]). We also cite a branch and bound solution scheme [5] based on the relaxation of the binary variables u_i.

The lack of general purpose SMIP solvers calls for the design of solution methods that fully exploit the special structure of the problem under analysis. Considering our model, we observe that if no strategic constraints are included the problem becomes separable into smaller optimization subproblems. Each of them, minimizes the objective function of a single unit on the entire time horizon. Even for long time horizon, the size of the subproblems is limited and the solution can be carried out by using the routines provided by commercial software packages (such as CPLEX and LINGO). When strategic constraints are considered decomposition can be again obtained by relaxing the linking constraints. Thus, a Lagrangian relaxation solution strategy may be applied. The design of such a strategy is the subject of a future research. Nevertheless, the inclusion of further details in this context would not germane to the experimental study provided in the sequel of this paper.

4. Numerical experience

In order to validate the proposed model we consider two different test problems. The first one consists of a small generation system and will serve as pilot experiment to fully show and comment the results. The second test problem is based on a more realistic generation system presented in [5].

In the first problem, we have simulated three units and a time horizon of four periods. Table 1 reports the main unit's characteristics. We assume that both the fixed and variable costs (f_i and C_i) are constant for all the time periods.

Furthermore, following [21], we assume that the seller does not bid exactly at each unit's variable cost, but at the bidding price of $(C_i + 2)$. This strategy allows to contribute to cover fixed costs.

Uncertainty is modelled as binary tree (see Fig. 4). In particular, at each stage, we assume that only two different clearing prices can be observed in the corresponding market. Thus, we have a total number of 32 scenarios. For all the time periods, we assume that the scenario tree has the same structure. In Table 2 we report the probability values (prob.) and the clearing prices (CP) associated to each node of the scenario tree, for the four time periods.

Table 1
Units' characteristics

Unit	f_i	C_i	Q_i	q_i
1	900	15	500	150
2	950	16	500	150
3	1000	18	500	150

Table 2
Probabilities distribution and clearing prices

Node	Period 1		Period 2		Period 3		Period 4	
	Prob.	CP	Prob.	CP	Prob.	CP	Prob.	CP
2	0.7	21	0.5	22	0.4	20	0.7	20
3	0.3	18	0.5	19	0.6	17	0.3	17
4	0.5	22	0.4	21	0.25	21	0.5	22
5	0.2	18	0.2	18	0.25	20	0.2	18
6	0.2	20	0.2	19	0.25	20	0.2	19
7	0.1	17	0.2	17	0.25	17	0.1	16
8	0.3	20	0.2	21	0.3	21	0.3	20
9	0.2	17	0.2	18	0.2	19	0.2	17
10	0.1	20	0.1	20	0.1	20	0.2	17
11	0.1	18	0.2	19	0.1	18	0.1	18
12	0.1	21	0.1	19	0.1	21	0.1	21
13	0.1	19	0.1	18	0.1	20	0.1	19
14	0.05	19	0.05	18	0.05	19	0.05	19
15	0.05	18	0.05	17	0.05	18	0.05	18

Table 3
Units' state

Unit	Period 1	Period 2	Period 3	Period 4
1	On	On	On	On
2	On	On	On	On
3	On	On	Off	Off

Table 4
Capacity allocation for the first period

s	DEM		AM		BM	
	Offer	Sell	Offer	Sell	Offer	Sell
1				500–500–0	0–0–0	0–0–0
2		0–0–500	500–500–0			0–0–0
3				500–250–0	0–250–0	0–250–0
4	0–0–500					0–125–0
5				0–0–250	500–500–250	500–500–250
6		0–0–0	0–0–500			500–500–0
7				0–0–0	500–500–500	500–500–0
8						500–250–0

On the basis of the clearing price that has been observed we can deduce, at each stage and for each unit, the percentage of quantity of energy sold with respect to that offered. When it happens to a unit to be a marginal one we assume that the percentage of energy sold is 0.5. Furthermore, we choose an opportunity cost equal to 5 for the unsuccessful quantity of energy at the end of the bidding process. Finally, for all the three units we have fixed the minimum up time D_i and down time d_i equal to 2.

The model has been implemented and solved using the solver LINGO 8.0 [22]. As explained in the previous section, if no strategic constraints are included the resulting basic model is separable with respect to the units. This feature makes the problem numerically tractable for moderate size and allows to solve both the problems by using the same software package.

Table 3 reports information about the state (on/off) of the units for the four time periods. In particular, we observe that the up/down time constraints force unit 3 to stay on (than off) for the minimum up (down) time periods.

The recommendations provided by the basic model for the four time periods are reported in Tables 4–7. Specifically, for each unit and scenario (s), both the amount of energy offered and sold are reported for the three auctions.

Let us analyze these results by focussing, for example, on the first period (Table 4). The results reported suggest to adopt the following strategy: in the DEM auction, the seller should offer 500 MW by using the third unit only. On the basis of his offer and the observed clearing prices, it may happen either that he sells all the offered amount (scenarios 1–4) or that his offer is not successful

Table 5
Capacity allocation for the second period

s	DEM		AM		BM	
	Offer	Sell	Offer	Sell	Offer	Sell
1				0–0–0	0–0–0	0–0–0
2		500–500–500	0–0–0			0–0–0
3				0–0–0	0–0–0	0–0–0
4	500–500–500					0–0–0
5				0–0–0	0–0–500	0–0–0
6		500–500–0	0–0–500			0–0–0
7				0–0–0	0–0–500	0–0–0
8						0–0–0

Table 6
Capacity allocation for the third period

s	DEM		AM		BM	
	Offer	Sell	Offer	Sell	Offer	Sell
1				0–0–0	500–500–0	500–500–0
2		0–0–0	0–0–0			500–500–0
3				0–0–0	500–500–0	500–500–0
4	0–0–0					500–250–0
5				500–500–0	0–0–0	0–0–0
6		0–0–0	500–500–0			0–0–0
7				250–0–0	250–500–0	250–500–0
8						250–250–0

(scenarios 5–8). The quantity to offer in the AM auction are reported in the fourth column of the same table. The model suggests different plans for the scenarios. In particular, for the first four scenarios it appears to be convenient to offer the remaining amount of energy in the AM auction, whereas for the other scenarios only the 500 MW of the third unit are offered again and the remaining capacity is preserved for the MB auction. Once again, by a simple comparison between the units' offer prices and the observed AM clearing price, we can deduce the quantities sold for each unit and for each possible event. In particular, we underline the fact that when a unit is marginal only a partial percentage is sold (column 5). The seller may decide now his offer for the BM auction (see the sixth column). The last column reports the amount effectively sold in the third market. We observe that the amount offered, but not sold represents for the seller a lost of profits. By increasing the opportunity cost, it is possible to penalize this circumstance, and the model will, consequently, tend not to offer necessarily the whole capacity of each unit.

The considerations made for the first period, can be similarly extended to the other periods. We observe that, because of minimum up and down constraints, unit 3 is never used in the third and fourth periods.

Table 7
Capacity allocation for the fourth period

s	DEM		AM		BM	
	Offer	Sell	Offer	Sell	Offer	Sell
1				500–500–0	0–0–0	0–0–0
2		0–0–0	500–500–0			0–0–0
3				500–250–0	0–0–0	0–0–0
4	0–0–0					0–0–0
5				0–0–0	500–500–0	500–500–0
6		0–0–0	0–0–0			500–500–0
7				0–0–0	500–500–0	500–500–0
8						500–250–0

Table 8
Units' characteristics

Unit	f_i	C_i	Q_i	q_i	D_i	d_i
1	1200	24	455	150	8	8
2	1200	26	455	150	8	8
3	950	25	130	20	5	5
4	900	25	130	20	5	5
5	750	24	162	25	6	6
6	600	24	80	20	3	3
7	650	23	85	25	3	3
8	550	20	55	10	1	1
9	550	20	55	10	1	1
10	550	20	55	10	1	1

Let us now introduce the second test problem. It is based on a slight modification of the 10-units system presented in [5]. Table 8 reports the main characteristics of the units.

As far as the clearing prices are concerned we have used the information made available by *Red Eléctrica*, the Spanish grid operator. This information consists in a large updated database of the exchanged energy and clearing prices for each hour and every day of several years of operation of the Spanish market.[4] Starting from that historical data we have generated the scenarios by using a simulation approach based on the bootstrapping technique [23]. The time horizon considered in this test problem has been fixed to a realistic extent of 24 periods and 192 scenarios have been simulated. By taking advantage from the separability of the model with respect to the units, it is possible to consider larger horizons. However, even for limited number of periods and scenarios the amount of data to handle is enormous which makes the presentation of the full input data or the

[4] Web site of Red Eléctrica de España: http://www.ree.es.

complete results exceeding any space that could be reserved to this paper.[5] On the basis of the numerical results, the following observations may be drawn:

- the results of this test problem are consistent with the expected strategy and confirm the behaviour of the model observed in the previous test problem;
- the model tries to define the optimal allocation plan that hedges against all the possible scenarios;
- again the main factor influencing the bidding strategy is represented by the uncertain clearing prices that affect directly which offers are successful and which are not;
- it never happens in this case that a unit is marginal and thus only two cases can occur: the total quantity offered is successful or none;
- units having small values of the minimum up and down times have a more "active" schedule.

5. Concluding remarks

In this paper, we presented a stochastic model that suggests a capacity allocation strategy in a multi-auction competitive market. The seller is considered a price taker with no ability to exercise market power through economic and physical withholding. The mathematical formulation proposed is basically a unit commitment-based model that covers a multi-period time horizon. Uncertainty related to the clearing auctions information is incorporated into the model explicitly by means of a scenario tree representation. The decision stages correspond to the different bidding moments referring to the same time period, and not, as traditionally assumed, the successive time periods referring to the same auction. We verified the effectiveness of the model on a first small test problem than on a realistic problem by exploiting the separability of the model and by using a commercial software package.

However, the complexity of the problem increases with the number of units and also with the number of time periods. Moreover, the size of the optimization problem grows linearly in the number of scenarios and exponentially in the number of stages (auctions). Hence, it is likely that the computational tractability becomes a major challenge in the application of the proposed methodology for large-scale systems specially if strategical constraints should be included. This aspect presents good opportunities for the development of computationally efficient specialized methods.

Another open question in this context is related to the design of sophisticated and more tailored methods for the scenario-tree generation. Existent literature in this context can be classified into two main categories [15]: generation schemes embedded in the solution procedures of stochastic programs [24–26], and approaches based on the control of some statistical properties of the underlying random process [27,28]. While some progress has been done in the statistical modelling of the demand of electricity [29], the ongoing liberalization has prompted the need to define novel statistical models to simulate electricity prices. Despite some distributional similarities, electricity prices are dramatically different from equity prices, and this makes the existing financial models of little use to the accurate representation of the price process in the electricity business. Recently, Knittel and Roberts [30]

[5] For reproducibility reasons, the authors can make available, on request, the Lingo model and the input data spreadsheet.

presented different methods for the electricity prices modelling but, as they underline, this research area is still in its infancy.

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