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Portfolio Optimization for the Electricity Traders in the Italian Market

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ABSTRACT

The paper considers the problem of maximizing the profits of a trader operating in the Italian electricity market. The problem consists in selecting the contracts portfolio and define the bidding strategy in the wholesales market while respecting the technical and regulatory constraints. A novel solution method based on a enhanced discovery of the search domain in the simulated annealing technique has been developed for its solution and a set of realistic test problems have been generated for its validation. The experimental results show that our method outperforms the standard simulated annealing by an improvement gap of 20.48% in average.

Keywords: Electricity Traders, Competitive Markets, Probabilistic Models, Heuristic Approach

1. Introduction

With the deregulation of the electricity business the energy price is not anymore defined by the regulator but subject to the market trading. As a result, both suppliers and consumers should construct their policies on the basis of the profit maximization as a main criteria in their participation in the electricity market. With this view, the electricity is seen as any commodity (gas, oil, …) that is subject to the demand/offer rule. However, the power system is characterized by many complicating features the most important of which is the fact that electricity not storable. Consequently, existing mathematical models for the profit optimization of other commodities can not be applied to the power system context and new models should be developed and solved. Such optimization problems become even more difficult whenever it concerns an operator that is allowed to both buy and sell in the same time. This is the case of the electricity trader, an operator that was introduced in Italy with the deregulation and whose role is to be an intermediary between the sellers and the buyers who are not allowed and/or not interested in participating in the wholesales market. The traders are allowed to buy and sell energy either through bilateral contracts or in the wholesales market. Moreover, the traders in the Italian market have also the opportunity to trade the so-called Green Certificates. These are an environmental conservation tool introduced by the deregulation Order 79/99 that obliges each producer to dispatch a fraction of his production from renewal resources. A Green Certificate, having a value of 100 MWh and released by the system operator, represents the proof of such a production that can be traded as contracts or in a specific market. It is worth noting here that the traders in the Italian system are not allowed to have an own production capacity or to have a role in the distribution business.

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In order to perform their mission, the traders have to interact with the following electricity actors (figure 1):

- **Producers**: traders can buy energy/capacity through bilateral contracts from either foreign suppliers or Italian producers having plants with a capacity of at least 10 MVA;
- **Consumers**: traders can sell energy/capacity through bilateral contracts to the so-called eligible consumers, a category that actually includes all the Italian consumers with the exception of the domestic customers;
- **GME/GRTN**: the wholesale market represents another opportunity for the traders to buy/sell energy, capacity, and green certificates. It is organized by the market operator **GME** (*Gestore del Mercato Elettrico*) with the technical support of the system operator **GRTN** (*Gestore di Rete di Trasmissione Nazionale*).

More specifically, while the GRTN has the key role of ensuring the security and the reliability of the system, the GME is responsible for managing the short-term forward electricity exchange by using the auction as a market model for the price definition. The GME collects the offers/bids submitted by the different players and provides the market clearing information. The market clearing price (MCP) and quantities (MCQ) for each time period are determined as the intersection of the aggregated supply and demand curves (Figure 2).

This paper has the objective of defining a profit maximization model for a trader operating in the Italian electricity market and also developing an heuristic method for its solution. The field of the optimal management of power systems has represented for long time a very active area of research. Besides the unit commitment [Car96, She94, Tak96, Val01], the optimal power flow [Bob94, Bun86, Cho90, Gud92, Now97] and capacity expansion [Mal94, Wu02] problems that have had big importance for the electricity management both before and after the deregulation, many new problems have been generated only with the advent of the deregulation. Some of such problems arise as a revision of old models to make them suitable for the competitive context, and many others describe the recent challenges that face the different operators and should be defined from scratch. The literature is quite rich for both kind of problems addressed in a deterministic or a stochastic framework. Examples of the first category include the optimization of the reservoirs in hydroelectric systems [Arc99, Fle97, Jac95, Now96], the management of the contingencies [Alv98, Men98].....etc, whereas the list of the new challenging problems is quite long since each operator is involved with his specific models. A non exhaustive list may include...
the auction clearing [Ber04, Bjo02] to be solved by the GME, the bidding strategy definition [Bai01, Gro00, Mar01, Tri05] and the contract selection [Con01, Fle98, Kay90, She04] to be solved by producers and consumers, and the energy pricing and tariff definition [Boh84, Bor02, Ish95, Tri05] that involve mainly the distributors. Many of these problems and others are discussed in recent books such as [Sha02, She99, Sto02] and their mathematical formulations are presented in the review produced by Conejo and Prieto [Con01]. This last paper includes also a very general discussion and a generic mathematical outline of the auction participation, self operation and contract selection from the traders’ viewpoint. To the best of our knowledge no further literature exists about this topic and no detailed formulation and/or experimental research has been carried out neither in the Italian context nor for any other electricity market worldwide.

Figure 2: Price definition in the electricity market

This paper addresses this problem and proposes an integrated mathematical framework for the trader’s profit maximization through not only the energy/capacity trading but also the green certificates management. The paper is organized as follows: in sections 2 we introduce a deterministic model, in section 3 its probabilistic counterpart and in section 4 we discuss a hybrid simulated annealing method for their solution. Section 5 will be devoted to the experimental results and some concluding remarks will be drawn in the last section.

2. Deterministic Model

In developing the portfolio optimization model for an Italian trader we consider only physical transactions without taking into account the financial tools. This is due to the fact that the financial transactions are still not well developed in the Italian market that has became operative only recently and, thus, it is still not clear which kind of financial tools will be adopted [Ter01]. As a consequence, in developing our model we should consider that any quantity of energy that has been approved in the marketplace must be really dispatched. Moreover, it is necessary to take into account the topology of the network. The electric system in Italy is, indeed, divided into six geographic zones that are connected by a limited transmission capacity causing often congestion problems (Figure 3). For this reason we will consider explicitly in the model the zonal index \( z \) whenever it is appropriate and specially in indicating the consumption/emission point to which each energy transaction refers. Such information will eventually be used by the GME for clearing the energy markets whenever congestion happens. More specifically, the market clearing price will be
unique if no congestion in the transmission network can be caused by the successful transactions, otherwise the GME defines different clearing prices one for each zone [Cer02]. A peculiarity of the Italian market consists in introducing a regulatory constraint imposing that, while zonal prices are allowed on the selling side, a uniform purchasing price has to be applied for all the zones of the Italian system for the first three year of market operation.

In what follows we present the description of the deterministic model on the basis of the operative opportunities that a generic trader has to make profits, i.e. bilateral contracts, wholesales market, and green certificates contracts. The problem is assumed to be separable with respect to time intervals and the models will be presented, thus, for a generic time period without specifying any index for it.

2.1. Bilateral Contracts
Bilateral contracts represent for the electricity operators and mainly for the traders a risk aversion tool against the market price volatility. A trader has, thus, always advantages to diversify his energy portfolio by designing a set of purchase and sales contracts. We assume here that the trader designs for each contract a number of alternatives that may differ in price and/or quantity. Among these alternatives the model selects the set of contracts that maximizes the trader’s profits on the basis of the expected market clearing price.

Once a contract has been selected the trader has the obligation to satisfy it. Otherwise, in case of its total or partial dissatisfaction, he will be subject to a penalty that is proportional to the missing quantity. Each bilateral contract will be, thus, totally characterized by the attributes price, quantity, and penalty. In order to formulate the constraints related to the bilateral constraints we need to introduce the following notation:

\[ Z \] set of the geographic zones
\[ CB^A \] set of purchase bilateral contracts
\[ CB^V \] set of sales bilateral contracts
\[ OPV^E \] set of alternatives for each energy sales contract
\[ OPA^E \] set of alternatives for each energy purchase contract
The decision variables corresponding to the bilateral contracts are:

- \( BINV_{hi} \): binary variable that takes value 1 if the alternative \( i \) of the sales bilateral contract \( h_z \) has been selected and 0 otherwise
- \( BINA_{jz} \): binary variable that takes value 1 if the alternative \( i' \) of the purchase bilateral contract \( j_z \) has been selected and 0 otherwise
- \( FV_{hi} \): fraction of energy satisfied out of the alternative \( i \) of the sales bilateral contract \( h_z \)
- \( FA_{jz} \): fraction of energy satisfied out of the alternative \( i' \) of the purchase bilateral contract \( j_z \)

The constraints on bilateral contracts can be written as:

1. \( FV_{hi} \leq BINV_{hi} \quad \forall h_z \in CB^i, \forall i \in OPV^E \)
2. \( FA_{jz} \leq BINA_{jz} \quad \forall j_z \in CB^A, \forall i' \in OPA^E \)
3. \( \sum_{i \in OPV^E} BINV_{hi} \leq 1 \quad \forall h_z \in CB^i \)
4. \( \sum_{i' \in OPA^E} BINA_{jz} \leq 1 \quad \forall j_z \in CB^A \)

Constraints (1) and (2) ensure that the fraction of energy effectively satisfied is different from zero only if the corresponding contract has been selected, whereas constraints (3) and (4) impose that at most one alternative is selected for each contract \( h_z \) or \( j_z \), respectively.

### 2.2. Wholesale Market

The wholesale electricity market consists in four different auctions [GME02]:

- **Day Ahead Market (DAM):** defines the preliminary dispatching programme. Operators participate by submitting energy supply offers and demand bids. Supply offers can be either simple i.e. constituted by a price/quantity pair or multiple i.e. formed by up to four block components with the associated prices forming a non decreasing function.
- **Adjustment market (AM):** defines an updated preliminary dispatching programme. In this energy market both sellers and buyers are allowed to adjust their day-ahead
schedules on the basis of the new information about the load forecast and the unit status.

- Dispatching Service Market (DSM): this market allows the definition of the final dispatching programme. Only sellers are allowed to bid for the provision of the spinning and non-spinning reserves.
- Green Certificates Market (GCM): this a weekly market that represents an occasion for the operators to exchange green certificates.

In order to simplify the model's description, we suppose that all the offers/bids are simple and we restrict our attention to only the DAM (MGP in Italian, i.e. “Mercato del Giorno Prima”). The extension to the other auctions is quite straightforward but involves additional variables and constraints without adding further insight or requiring any additional analytical developments for the application. The formulation of the constraints related to the DAM requires the introduction the following notation:

$I$ set of emission points for which the trader is allowed to present offers

$P$ set of consumption points for which the trader is allowed to present bids

$NV_{MGP}$ set of supply offers presented at the DAM

$NA_{MGP}$ set of purchasing bids presented at the DAM

$L_{ij}$ energy transmission limit between the zones $i$ and $j$ before the DAM (MW)

$P_{MGP}$ DAM clearing price in zone $z \in Z$ (€/MWh)

$PUN$ uniform purchasing price in the DAM auction calculated as the weighted average of the zonal clearing prices (€/MWh)

$c_z$ contribution of zone $z$ in forming the $PUN$

$U_{iz}^{MGP}$ up margin available for the DAM in the emission point $i_z$ in zone $z$ (MW)

$DW_{pz}^{MGP}$ down margin available for the DAM in the consumption point $p_z$ in zone $z$ (MW)

$Q_{MGP}^{\min}$ minimum quantity of energy that can be exchanged in the DAM (MW)

The decisions to be taken by the trader are expressed by means of the variables:

$QV_{h_{iz}}^{i_z}$ quantity of energy to be specified in the supply offer $h_{GMP}$ to the DAM referring to the emission point $i_z$ located in zone $z$ (MWh)

$PV_{h_{iz}}^{i_z}$ selling price to be specified in the supply offer $h_{GMP}$ to the DAM referring to the emission point $i_z$ located in zone $z$ (€/MWh)

$QA_{j_{pz}}^{p_z}$ quantity of energy to be specified in the purchasing bid $j_{GMP}$ to the DAM referring to the consumption point $p_z$ located in zone $z$ (MWh)

$PA_{j_{pz}}^{p_z}$ purchasing price to be specified in the bid $j_{GMP}$ to the DAM referring to the consumption point $p_z$ located in zone $z$ (€/MWh)

$F_{ij}^{MGP}$ energy transmission flow between zones $i$ and $j$ resulting from the offers/bids presented to the DAM (MW)

The constraints can be written thus as:

\[ (5) \quad \sum_{h_{iz}} QV_{h_{iz}}^{i_z} \leq U_{iz}^{MGP}, \forall h_{MGP} \in NV_{MGP}, \forall i_z \in I, \forall z \in Z \]

\[ (6) \quad \sum_{j_{pz}} QA_{j_{pz}}^{p_z} \leq DW_{pz}^{MGP}, \forall j_{MGP} \in NA_{MGP}, \forall p_z \in P, \forall z \in Z \]
Constraints (5) and (6) ensure the respect of the up/down margins of the units installed in the emission/consumption points. Constraints (7) and (8) impose a minimum energy quantity that can be exchanged in the DAM. Constraints (9) and (10) are the conditions on the offering/bidding price in order to be from one side economically feasible and from the other successful in the DAM. More specifically, a selling price referring to an emission point located in zone \( z \) will be economically feasible if it is higher than a minimum bidding price \( P_{\text{min}} \) to be chosen by the trader and will be successful only if it is less than the clearing price of the same zone. Analogously, a purchasing price referring to a consumption point located in zone \( z \) will be feasible if it is less than a trader’s specified maximum price \( P_{\text{max}} \) and will be successful only if it is bigger than the uniform purchasing price. The last set of constraints (11) ensure the respect of the transmission limit between each pair of adjacent zones. The variables \( F_{ij}^{\text{MGP}} \) represent the total imbalance between the energy sold and purchased between each pair of zones \( i \) and \( j \). Such variables are not thus independent variables since they can be expressed in terms of the quantities of energy exchanged in the DAM.

2.3. Green Certificates Contracts
Besides the weekly market organized by the GME, the green certificates (\( CV \) in Italian, i.e. “Certificati Verdi”) can be exchanged through bilateral contracts giving another opportunity to the traders to make profits. In order to model the constraints related to these contracts we introduce the following additional notation:

- \( CBA^{CV} \) set of green certificate sales contracts
- \( CBI^{CV} \) set of green certificate purchase contracts
- \( OPV^{CV} \) set of alternatives for a green certificate sales contract
- \( OPA^{CV} \) set of alternatives for a green certificate purchase contract
- \( PRV_{kCV}^{i} \) selling price of the alternative \( i \) of the green certificate sales contract \( k_{CV} \) (€/MWh)
- \( NOV_{kCV}^{i} \) number of green certificates agreed in the alternative \( i \) for the sales contract \( k_{CV} \)
- \( PRA_{wCV}^{i} \) purchasing price of the alternative \( i' \) of the green certificate purchase contract \( w_{CV} \) (€/MWh)
- \( NOA_{wCV}^{i} \) number of certificates agreed in the alternative \( i' \) for the purchase contract \( w_{CV} \)

Analogously to the energy bilateral contracts, the decision variables here are:

- \( BINV_{kCV}^{i} \) binary variable that takes value 1 if the alternative \( i \) of the green certificate sales contract \( k_{CV} \) has been selected and 0 otherwise
- \( BINA_{wCV}^{i} \) binary variable that takes value 1 if the alternative \( i' \) of the green certificate purchase contract \( w_{CV} \) has been selected and 0 otherwise
The conditions that the traders should respect when trading green certificate contracts are the following:

\[(12) \quad \sum_{i \in OPV^{CV}} BINV^{CV}_{k_{CV}} \leq 1 \quad \forall k_{CV} \in CBV^{CV}\]

\[(13) \quad \sum_{i' \in OPV^{CV}} BINA^{CV}_{w_{CV}} \leq 1 \quad \forall w_{CV} \in CBA^{CV}\]

\[(14) \quad \sum_{i' \in w_{CV}} NOA^{CV}_{w_{CV}} * BINA^{CV}_{w_{CV}} = \sum_{i \in k_{CV}} NOV^{CV}_{k_{CV}} * BINV^{CV}_{k_{CV}}\]

Analogously to the bilateral contracts, constraints (12) and (13) ensure that at most one alternative is selected for each sales/purchase contract. Constraint (14) matches the number of sold and purchased green certificates exchanged.

Finally, the whole model should include the balance constraint:

\[(16) \quad \sum_{j_s} QOA^{j_s}_{j_s} * FA^{j_s}_{j_s} + \sum_{p_i,j} QA^{p_i}_{j_{s, d}} = \sum_{h_i} QOV^{h_i}_{h_i} * FA^{h_i}_{h_i} + \sum_{i\in h_i} QV^{i}_{h_{i, d}}\]

that matches the total energy sold and that purchased by the trader either through bilateral contracts or in the DAM.

2.4. Objective Function

The objective of the trader is that of maximizing the profits defined as the difference between the revenues and the costs.

\[
\begin{align*}
\text{Max} & \quad \sum_{h_i,j_s} \left[ PRV^{h_i}_{h_i} * QOV^{h_i}_{h_i} * FA^{h_i}_{h_i} - PNV^{h_i}_{h_i} * QOV^{h_i}_{h_i} \right] - \\
& - \sum_{j_s} \left[ PRA^{j_s}_{j_s} * QOA^{j_s}_{j_s} * NOV^{j_s}_{j_s} * PUN^{j_s} \right] + \sum_{p_{i,j}} QA^{p_i}_{j_{s, d}}] + \\
& \sum_{k_{CV},j_s} \left[ PRV^{k_{CV}}_{k_{CV}} * NOV^{k_{CV}}_{k_{CV}} * BINV^{k_{CV}}_{k_{CV}} \right] - \sum_{w_{CV},j_s} \left[ PRA^{w_{CV}}_{w_{CV}} * NOA^{w_{CV}}_{w_{CV}} * BINA^{w_{CV}}_{w_{CV}} \right]
\end{align*}
\]

The first summation refers to the revenues deriving from the sales of the energy bilateral contracts minus eventual penalty for any dissatisfied fraction. The second summation refers to the cost of buying energy through bilateral contracts to which we add eventual penalty for dissatisfied fraction of any contract. The third summation represents the profits that may result from the exchange of energy in the wholesales market. The last two summations represents the difference between the revenues deriving from the sales of the green certificates and the costs to be supported by the trader for their purchases through bilateral contracts.
In the previous model, the zonal clearing prices and consequently the uniform purchasing price have been considered as known quantities. However, at the moment of presenting the offers/bids the trader does not know the exact value of these prices and, thus, their probabilistic representation is necessary in order to have a more realistic formulation [Pre93]. For this reason, we assume that the zonal clearing prices can be represented as independent random variables having a normal distribution with mean value $\eta_z$ and variance $\sigma_z^2$. Since the uniform purchasing price is a linear combination of the zonal prices, it will be also represented as a normal random distribution with a mean and variance calculated as the linear combination of the mean values and the variances of the zonal prices, respectively [BB80].

The model that we propose here is based on a chance constrained formulation: instead of imposing that the constraints (13)-(14) are satisfied for all the possible realizations of the random variables involved, we relax them by accepting that each of these constraints can be violated by a probability that does not exceed a chosen value $\alpha$. Mathematically, such conditions can be expressed as:

$$
(13.1) \quad P\left[ PA_{\text{JMS}}^p \geq PUN, \forall j_{\text{MGP}} \in NA_{\text{MGP}}, \forall p_z \in P, \forall z \in Z \right] \geq 1 - \alpha
$$

$$
(14.1) \quad P\left[ PV_{\text{hMS}}^i \leq P_{\text{MGP}}, \forall h_{\text{MGP}} \in NV_{\text{MGP}}, \forall i_z \in I, \forall z \in Z \right] \geq 1 - \alpha
$$

This representation imposes a joint probability level $p_{\text{target}} = (1-\alpha)$ on the satisfaction of the price constraints over the whole set of constraints. The profit maximization model with constraints (13.1) and (14.1) is a probabilistic formulation and for its solution we need to develop its deterministic equivalent version. We focus here on the constraint (13.1) but the extension of the results presented here for constraint (14.1) is straightforward.

Let $\xi$ denote the vector of all equal $n = |NA_{\text{MGP}}|*|P|$ components consisting in the independent random variables having a normal distribution. The mean and standard deviation vectors of the $PUN$ are composed thus of $n$ times the values $\mu$ and $\sigma$, respectively. The constraint (13.1) could be written as:

$$
P[A] \geq 1 - \alpha
$$

where $A$ represents the event:

$$
A = \{ PA_{\text{JMS}}^p \geq \xi \} \quad \text{and his complementary is} \quad A^C = \{ PA_{\text{JMS}}^p < \xi \}
$$

By using the properties of the probability, we can write the stochastic constraint in the following equivalent way:

$$
P[A] = 1 - P[A^C] \geq 1 - \alpha \quad \Rightarrow \quad P[A^C] = P[PA_{\text{JMS}}^p < \xi] \leq \alpha
$$

Now let indicate by $F_{\xi}$ the cumulative distribution function (CDF) of the random vector $\xi$, then from the probability theory it is known that:

$$
P[PA_{\text{JMS}}^p \geq \xi] = F_{\xi}(PA_{\text{JMS}}^p) \quad \Rightarrow \quad P[PA_{\text{JMS}}^p < \xi] = 1 - F_{\xi}(PA_{\text{JMS}}^p)
$$

Thus the probabilistic constraint can be expressed as:

$$
1 - F_{\xi}(PA_{\text{JMS}}^p) \leq \alpha
$$

or equivalently, by using the properties of the CDF normal standard that we denote by $\Phi_\xi$, as:

$$
1 - \Phi_{\xi}\left( \frac{PA_{\text{JMS}}^p - \mu}{\sigma} \right) \leq \alpha \quad \forall j_{\text{MGP}} \in NA_{\text{MGP}}, \forall p_z \in P, \forall z \in Z
$$
or, by reordering the terms, as:
\[
\Phi_{\xi} \left( \frac{PA_{j,\text{hosp}}^z - \mu}{\sigma} \right) \geq 1 - \alpha \quad \forall j_{\text{MGP}} \in NA_{\text{MGP}}, \forall p_z \in P, \forall z \in Z .
\]

Now we are able to define the deterministic equivalent of the probabilistic constraint (13.1) that can be written as:
(13.2) \[
PA_{j,\text{hosp}}^z \geq \mu + R_{(1-\alpha)} \sigma = \mu + r \sigma \quad \forall j_{\text{MGP}} \in NA_{\text{MGP}}, \forall p_z \in P, \forall z \in Z
\]
in which \( R_{(1-\alpha)} = r \) represents the value, called r-value, in correspondence of which the normal standard distribution \( \Phi_{\xi} \) takes the probability value \((1-\alpha)\).

With this representation the profit maximization model, with constraints (13.2) substituting constraint (13.1), is a deterministic equivalent formulation depending on the r-value and that could be solved by conventional optimization methods in order to determine the optimal values of the variables \( PA_{j,\text{hosp}}^z \). Such values will be then used to compute the probability \( P [PA_{j,\text{hosp}}^z \geq \xi] \) i.e. the CDF, in order to check whether the stochastic constraint (13.1) is satisfied with the desired level of probability. For this purpose we use a numerical method based on the multivariate integration technique of Genz to calculate the CDF value, that we denote by \( p \), corresponding to the current solution [Gen92]. The solution will be considered acceptable only if the difference between the CDF value \( p \) and the desired probability \( p_{\text{target}} \) is below a small tolerance value \( \varepsilon \), that is \( | p - p_{\text{target}} | \leq \varepsilon \).

Otherwise, if the latter condition is not satisfied then it will be necessary to determine a new value of the parameter \( r \) and solve again the deterministic equivalent problem with an updated constrains (13.2). The procedure is repeated till the satisfaction of the target probability level.

We shall note that, by introducing the stochasticity into the model, the objective function will be slightly different from the one presented in the previous section. Specifically, the term:
\[
\sum_z \left[ p_z^{\text{MGP}} \sum_{i,z,\text{hosp}} QV_{i,z}^{l} - PUN \sum_{p_z,\text{hosp}} QA_{j,\text{hosp}}^p \right]
\]
should be substituted by the term:
\[
\sum_z \left[ \eta_z \sum_{i,z,\text{hosp}} QV_{i,z}^{l} - \mu \sum_{p_z,\text{hosp}} QA_{j,\text{hosp}}^p \right]
\]
where \( \eta_z \) indicates the mean value of the zonal price \( z \) and \( \mu \) represents the mean value of the uniform purchasing price \( PUN \), with \( \mu = \eta_1 + \eta_2 + \ldots + \eta_z \) as previously.

To conclude this section we describe below the algorithm that allows us to update the multivariate r-value [Ozt03]:

1. Find the univariate r-value corresponding to the probability level \( p_{\text{target}} = (1-\alpha) \) by using the following approximation due to Abramowitz and Stegun [AS64]:
\[
r = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \varepsilon (1-\alpha) \quad \text{with} \quad t = \sqrt[3]{\frac{\ln \frac{1}{1-(1-\alpha)^3}}{1-(1-\alpha)^3}}
\]
where the error \( | \varepsilon (1-\alpha) | < 4.5 \times 10^{-4} \) can be considered negligible.

The value of the constants used are the following:
\[
c_0 = 2.515517 \quad d_1 = 1.432788
\]
\[
c_1 = 0.802853 \quad d_2 = 0.189269
\]
\[
c_2 = 0.010328 \quad d_3 = 0.001308
\]

Set \( r = r_{\text{target}} = r_{\text{lower}} \).
2. Set $r_{upper}$ to a sufficiently big starting value.
3. Solve the profit maximization problem and compute the corresponding CDF value. By using the approximation of step 1 find the univariate $r$-value $r = r_2$.
4. Solve again the profit maximization problem using the $r$-value $r_{lower}$ and compute the CDF value. Use the approximation of step 1 in order to compute the univariate $r$-value $r = r_1$.
5. Compute the $r$-value $r_{new}$ as follows:
   \[ r_{new} = r_{lower} + \frac{(r_{target} - r_1)}{(r_2 - r_1)}(r_{upper} - r_{lower}) \]
6. Solve again the profit maximization problem using $r_{new}$ and compute the corresponding CDF value, to be denoted by $p$, and then the univariate $r$-value $r = r_{temp}$ by using the approximation of step 1.
7. If $|p - p_{target}| \leq \epsilon$ the algorithm terminates, otherwise go to step 8.
8. If $r_{temp} \leq r_{target}$ then set $r_1 = r_{temp}$ and $r_{lower} = r_{new}$, otherwise set $r_2 = r_{temp}$ and $r_{lower} = r_{new}$.
9. Go to step 5.

The first four steps are, indeed, the initialization part of the algorithm. The update of the $r$-value happens at steps 5 and 8 on the basis of a linear interpolation rule. The algorithm is stopped in step 7 as soon as the desired probability level is reached.

4. Solution Method

For non trivial applications the deterministic equivalent model of the profit maximization problem is characterized by a high number of variables and constraints. The dimension of the problem increases, indeed, with the number of contracts, auctions, offers and bids, etc. Solving, thus, the problem with exact solvers is generally incompatible with the timings of the market requirements. For this reason we developed a heuristic algorithm based on a variant of the simulated annealing (SA) method. The new heuristic, called hybrid simulated annealing method, consists in performing a better exploration of the neighborhood of the current solution. Before describing our method we present first the standard SA in order to fix the notation and to emphasize later the difference between the two variants.

Simulated Annealing Algorithm:
1. Select a starting solution $current_zeta$.
2. Define a search neighborhood of the starting solution.
3. Select a starting value of the temperature $T > 0$.
4. While the freezing temperature is not reached, the maximum number of iterations, and the maximum execution time are not reached:
   5. Generate randomly a new solution $new_zeta$ in the neighborhood of $current_zeta$.
   6. Compute $\Delta_zeta = new_zeta - current_zeta$.
   7. If $\Delta_zeta > 0$ then accept the new solution by setting $current_zeta = new_zeta$.
   8. If $\Delta_zeta \leq 0$ then accept the new solution with probability $P(\Delta_zeta) = e^{\Delta_zeta/T}$.
9. Reduce the value of $T$ by the cooling ratio.
10. Return the best solution reached.

This general algorithm can be applied to our profit maximization problem in the following way. The starting solution in step 1 can be determined by selecting arbitrarily one or more selling bilateral contract and one or more purchasing contract that ensure the feasibility of the solution. The definition of a search neighborhood in steps 2 and 6 consists in selecting
randomly one alternative of a selling or a purchasing contract and change its status, i.e. set its corresponding decision variable to one if it was zero, and vice-versa.

Even though the SA technique has shown to be very efficient in the solution of the trader’s optimization problem, we have proposed and implemented a novel variant, called hybrid SA, that attempts, at each iteration, to explore in a better way the neighborhood of the current solution. The variant consists in generating randomly, in the neighborhood of current_zeta, not just one solution but a sample of, say, k solutions that is sufficiently representative of the neighborhood. Among the generated solutions we select the one having the highest objective function that we denote by best_value. On the other hand, we calculate the mean value m_{cur} and the standard deviation \sigma_{cur} of the k solutions and we assume that they follow a normal distribution. The difference between a function of these values \text{f}(m_{cur}, \sigma_{cur}) and the best_value will be then used in our implementation as measure of significativity (representativity) of the generated sample. If such a difference is higher than a given threshold than the sample is considered not sufficiently representative of the neighborhood and the generation of an additional number of k’ solutions is necessary. The process is repeated till the satisfaction of the threshold on the difference \text{[f}(m_{cur}, \sigma_{cur}) - best_value]. We set thus new_zeta = best_value and the hybrid SA algorithm continues following the standard SA iterative scheme.

The efficiency of the method clearly depends on the choice of the function \text{f}(m_{cur}, \sigma_{cur}) and that of the threshold value. Several functions \text{f}(m_{cur}, \sigma_{cur}) could be used such as for example that corresponding to the value m_{cur} + 2 * \sigma_{cur}, whereas the value to be assumed by the threshold depends on the cardinality of the set of solutions that could be generated randomly and, consequently, on the problem’s dimension. Such a value, that will be determined empirically as function of the number of the problem’s variables, will represent a trade-off between the necessity of generating a significant sample of each search domain and that of not spending a high amount of time in the solution of the generated problems.

5. Computational Experiments

Both the standard and the hybrid SA algorithms have been implemented in C language and by making use of ILOG CPLEX© 8.1 for the solution of the MILP problems. The computation of the CDF values has been performed with the help of a Matlab function that implements the Genz’s technique [Gen92]. The value of \epsilon used for the determination of the r-value depends on the target probability level (1-\alpha). For example the value (1-\alpha)=0.8 corresponds to \epsilon=0.005, the value (1-\alpha)=0.999 corresponds to \epsilon=0.0005, etc. A starting temperature value of \text{T} = 40.000 has been defined empirically that then decreases every 100 iterations of the algorithms with a cooling rate of 0.8. The number of solutions to be generated in the search domain at each iteration of the hybrid SA algorithm has been set to k = 10 and whenever the sample does not result to be significant an additional number of k’ = 5 solutions will be generated. The threshold value measuring the sample significativity has been set to 5% for test problems with up to 20.000 variables, to 8% for up to 50.000 variable and to 10% for a higher number of decision variables. Finally, a penalty price corresponding to a further loss of 30% of the contract price has been used.

In order to validate the profit maximization model and to test the performance of our solution method we have generated 36 test problems simulating the behaviors of a trader operating in the Italian electricity market by establishing bilateral and green certificate contracts and by participating in the DAM auction. The input data has been collected in such a way that the test problems could realistically represent the Italian applicative context. Indeed, most of these data have been collected from the information provided by the GRTN and the GME referring to the first months of operation of the Italian market.
The missing data has been collected from the Nord Pool which is characterized by a similar jurisdiction with respect to the Italian market.

Table 1. Size of the test problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Variables</th>
<th>Constraints</th>
<th>Binary Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>P 1</td>
<td>2.480</td>
<td>1.209</td>
<td>1.300</td>
</tr>
<tr>
<td>P 2</td>
<td>5.300</td>
<td>2.629</td>
<td>1.050</td>
</tr>
<tr>
<td>P 3</td>
<td>1.780</td>
<td>859</td>
<td>350</td>
</tr>
<tr>
<td>P 4</td>
<td>3.020</td>
<td>1.489</td>
<td>650</td>
</tr>
<tr>
<td>P 5</td>
<td>10.200</td>
<td>5.269</td>
<td>3.000</td>
</tr>
<tr>
<td>P 6</td>
<td>7.400</td>
<td>3.679</td>
<td>2.100</td>
</tr>
<tr>
<td>P 7</td>
<td>10.600</td>
<td>5.319</td>
<td>700</td>
</tr>
<tr>
<td>P 8</td>
<td>7.800</td>
<td>3.729</td>
<td>2.100</td>
</tr>
<tr>
<td>P 9</td>
<td>9.400</td>
<td>4.719</td>
<td>2.400</td>
</tr>
<tr>
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<td>10.200</td>
<td>5.099</td>
<td>2.800</td>
</tr>
<tr>
<td>P 11</td>
<td>11.000</td>
<td>5.393</td>
<td>3.200</td>
</tr>
<tr>
<td>P 12</td>
<td>11.800</td>
<td>5.979</td>
<td>3.600</td>
</tr>
<tr>
<td>P 13</td>
<td>12.600</td>
<td>6.419</td>
<td>4.000</td>
</tr>
<tr>
<td>P 14</td>
<td>13.400</td>
<td>6.859</td>
<td>4.400</td>
</tr>
<tr>
<td>P 15</td>
<td>14.200</td>
<td>7.299</td>
<td>4.800</td>
</tr>
<tr>
<td>P 16</td>
<td>14.680</td>
<td>7.359</td>
<td>4.800</td>
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<tr>
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<td>15.000</td>
<td>7.399</td>
<td>4.800</td>
</tr>
<tr>
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<td>16.200</td>
<td>8.079</td>
<td>5.200</td>
</tr>
<tr>
<td>P 19</td>
<td>17.400</td>
<td>8.759</td>
<td>5.600</td>
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<tr>
<td>P 20</td>
<td>18.900</td>
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</tr>
<tr>
<td>P 21</td>
<td>19.700</td>
<td>9.709</td>
<td>6.100</td>
</tr>
<tr>
<td>P 22</td>
<td>21.700</td>
<td>10.809</td>
<td>6.100</td>
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<tr>
<td>P 23</td>
<td>23.000</td>
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<td>30.500</td>
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<tr>
<td>P 29</td>
<td>38.100</td>
<td>18.109</td>
<td>11.100</td>
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<tr>
<td>P 30</td>
<td>45.700</td>
<td>21.609</td>
<td>13.100</td>
</tr>
<tr>
<td>P 31</td>
<td>53.300</td>
<td>25.109</td>
<td>15.100</td>
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<td>P 32</td>
<td>60.900</td>
<td>28.609</td>
<td>17.100</td>
</tr>
<tr>
<td>P 33</td>
<td>68.500</td>
<td>32.109</td>
<td>19.100</td>
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<td>P 34</td>
<td>76.100</td>
<td>35.609</td>
<td>21.100</td>
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<td>P 35</td>
<td>83.700</td>
<td>39.109</td>
<td>23.100</td>
</tr>
<tr>
<td>P 36</td>
<td>93.700</td>
<td>46.109</td>
<td>24.000</td>
</tr>
</tbody>
</table>

Table 1 reports the characteristics of our test problems on the basis of their size. For each test problem we report the total number of variables, the number of constraints, and the number of binary variables obtained as the sum of selling and purchasing contracts multiplied by the corresponding number of options. The number of selling/purchasing contracts considered varies from 35 to 1200 and the number of options goes from 5 to 10.
Table 2. Hybrid vs. standard SA after 20 minutes of execution time

<table>
<thead>
<tr>
<th>Problem</th>
<th>Standard SA</th>
<th>Hybrid SA</th>
<th>Improvement gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>P 1</td>
<td>202.159,7</td>
<td>412.018,0</td>
<td>50,9</td>
</tr>
<tr>
<td>P 2</td>
<td>63.988,3</td>
<td>141.271,8</td>
<td>54,7</td>
</tr>
<tr>
<td>P 3</td>
<td>33.058,4</td>
<td>45.890,5</td>
<td>39,7</td>
</tr>
<tr>
<td>P 4</td>
<td>84.125,7</td>
<td>157.094,7</td>
<td>46,3</td>
</tr>
<tr>
<td>P 5</td>
<td>126.256,9</td>
<td>255.876,3</td>
<td>150,6</td>
</tr>
<tr>
<td>P 6</td>
<td>441.265,3</td>
<td>506.989,0</td>
<td>12,9</td>
</tr>
<tr>
<td>P 7</td>
<td>126.256,9</td>
<td>143.275,7</td>
<td>11,8</td>
</tr>
<tr>
<td>P 8</td>
<td>453.526,2</td>
<td>631.619,6</td>
<td>28,1</td>
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<tr>
<td>P 9</td>
<td>119.788,9</td>
<td>146.262,9</td>
<td>18,1</td>
</tr>
<tr>
<td>P 10</td>
<td>121.860,5</td>
<td>182.334,4</td>
<td>33,1</td>
</tr>
<tr>
<td>P 11</td>
<td>146.320,1</td>
<td>199.656,3</td>
<td>26,7</td>
</tr>
<tr>
<td>P 12</td>
<td>186.540,0</td>
<td>199.960,2</td>
<td>15,7</td>
</tr>
<tr>
<td>P 13</td>
<td>194.626,1</td>
<td>228.663,3</td>
<td>14,8</td>
</tr>
<tr>
<td>P 14</td>
<td>202.360,1</td>
<td>269.336,3</td>
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<td>P 15</td>
<td>220.113,3</td>
<td>270.665,3</td>
<td>18,6</td>
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<tr>
<td>P 16</td>
<td>200.454,1</td>
<td>281.669,0</td>
<td>28,8</td>
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<tr>
<td>P 17</td>
<td>210.447,5</td>
<td>300.500,0</td>
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<td>P 18</td>
<td>230.012,0</td>
<td>312.447,1</td>
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<td>P 19</td>
<td>231.255,0</td>
<td>288.900,1</td>
<td>19,9</td>
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<td>P 20</td>
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<td>299.663,2</td>
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<tr>
<td>P 21</td>
<td>312.336,8</td>
<td>366.884,1</td>
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<tr>
<td>P 22</td>
<td>318.006,3</td>
<td>390.001,0</td>
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<td>325.994,0</td>
<td>387.446,2</td>
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<td>P 24</td>
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<td>15,8</td>
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<td>P 25</td>
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<td>331.595,3</td>
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<td>P 27</td>
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<td>P 28</td>
<td>494.662,3</td>
<td>530.663,1</td>
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<td>P 29</td>
<td>390.556,2</td>
<td>460.532,0</td>
<td>15,1</td>
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<td>P 30</td>
<td>540.232,0</td>
<td>579.994,0</td>
<td>6,8</td>
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<td>P 31</td>
<td>598.663,0</td>
<td>610.230,4</td>
<td>1,8</td>
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<td>P 32</td>
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<td>650.230,4</td>
<td>9,1</td>
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<tr>
<td>P 33</td>
<td>640.008,9</td>
<td>686.451,2</td>
<td>6,7</td>
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<tr>
<td>P 34</td>
<td>700.400,0</td>
<td>760.552,1</td>
<td>7,9</td>
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<tr>
<td>P 35</td>
<td>840.991,2</td>
<td>885.996,3</td>
<td>5,0</td>
</tr>
<tr>
<td>P 36</td>
<td>865.442,1</td>
<td>900.520,1</td>
<td>3,8</td>
</tr>
</tbody>
</table>

The experimental results reported in table 2 show, from one side the validity of the profit maximization model and from the other side the superiority of the hybrid SA method with respect to its standard counterpart. For all the test problems, indeed, the quality of the solution obtained by using our method is remarkably higher than that obtained with the standard SA when both are executed for the same amount of time (20 minutes) and for the same starting parameters. The improvement gap goes from a minimum value of 1.8% to a maximum of 54.7% with an average value of 20.48% over the 36 test problems. Similar behaviors have been observed for higher amounts of execution time. This confirms that even though our method is forced to generate, at each iteration, a higher number of problems it
is able to take advantage from discovering better the domain search in order to find a better solution. However, it is clear from the results that as the size of the problems increases the gap becomes less important and the time consumed in the solution of the generated problems penalizes, thus, the performance of the hybrid SA method. Moreover, it is worthwhile noting that the solution quality depends on the choice of the starting solution \( \text{current}_\text{zeta} \). Nevertheless, this choice affects only the best objective value to be reached by each of the two methods and does not influence the improvement gap (è vero?). The best objective value for both the techniques can be improved by defining a starting solution that is as close as possible to the optimal solution. One possibility to achieve this aim is to use the warm starting technique.

6. Conclusions

The objective of this paper is to offer to the traders operating in the Italian electricity market a decision support for their short and medium term activities. First we analyzed the role of the traders and their interaction with the other operators in the Italian business context in order to define the profits opportunities behind establishing bilateral contracts and participating in the wholesales market. We proposed then a deterministic profit maximization model that ensures the contracts selection and the bidding strategy definition. Since the market clearing prices are not known before the bidding we model them as random variables and we provided a chance constrained formulation that allows the prices constraints to be violated by a probability that does not exceed a chosen value \( \alpha \). For the solution of the deterministic equivalent of the probabilistic formulation we developed an heuristic method that efficiently generates multiple solutions within the search domain of the standard simulated annealing technique. Experimental results on realistic test problems have proved the validity of the profit maximization model and the superiority of the proposed method with respect to the standard SA algorithm. The improvement gap reaches a maximum value of 54.7% for small- and medium-scale problems that decreases for larger problems reaching a value of 1.8%. The average improvement of our method with respect to the standard SA algorithm is 20.48% over the 36 generated problems. Many improvements for both the optimization model and the solution method are possible. The model can be enhanced by including the financial contracts that ensure an efficient management of the trader’s aversion towards the risk, whereas the performance of the hybrid SA method can be improved by adopting a warm starting strategy.

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