OPTIMAL ROUTING AND RESOURCE ALLOCATION IN AD-HOC NETWORKS

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Abstract. This paper addresses the problem of simultaneously optimizing power consumption and routing in ad-hoc networks while ensuring the satisfaction of the required transmission demands between given origin-destination pairs. We devise a linear programming model for the selection of optimal routes and transmission schemes where the power and link capacity levels are suitably discretized. Since the constraint matrix of such a model contains a huge number of columns, we propose an exact algorithm for its solution based on a column generation approach. The associated column generation procedure is based on the solution of linear integer programming models. We report some computational results on networks with up to 65 nodes, showing the feasibility of our solution approach.

keywords: ad-hoc networks, power and capacity allocation, optimal routing, column generation.

1. Introduction

A wireless ad-hoc network consists of a set of devices (nodes) that exchange data flows via radio links. Each device has a limited transmission range. Two devices can communicate directly (single hop connection) if their distance is at most equal to their transmission range. On the other hand, if their distance is greater than their transmission range, they can communicate by using other devices as intermediate repeaters (multi-hop connection).

The main advantage of ad-hoc networks is that their use does not require any pre-installed infrastructure. For this reason this kind of wireless networks is widely used in all those contexts in which the installation of fixed devices is not economically and/or technically feasible. Because of such flexibility the interest in ad-hoc networks has recently increased generating several optimization problems challenging both the industry and the scientific communities.

Yuan et al. in [1] have dealt with the problem of throughput optimization and they have proposed a dual decomposition approach to solve the resulting convex model. Wang and Kar have addressed in [2] the rate-control problem by developing two different algorithms, one based on the primal approach and the other on its dual counterpart. The same problem has been considered by Kelly et al. that have focused on the issues of stability and fairness of the solutions [3].

Värbrand et al. have dealt in [4] with the resource allocation problem within a TDMA scheme and then with the frame scheduling problem within an STDMA scheme in [5] by implementing for each problem a specialized column generation method.

Toumpis and Goldsmith have also considered the capacity region problem for several transmission protocols [6]. The above mentioned approaches express the capacity of the network as a point in the convex hull of a finite number of feasible capacity allocations.

More complex problems that combine two or more aspects of the wireless networks management have also been introduced in the literature by several authors. Capone et al. have tackled the challenging problem of guaranteeing the quality of service (QoS), expressed in terms of bandwidth, within a flow maximization problem [7]. The problem of simultaneously optimizing scheduling, packet routing and power control has been addressed by Cruz and Santhanam in [8] and by Radunovic and Le Boudec in [9]. Analogously, Elbatt and Ephremides have proposed an optimal scheduling scheme that prevents strong levels of interference by means of a power control approach [10]. Johansson et al. have proposed original mathematical models that combine the routing optimization with the resources allocation [11], [12].

The solution methods proposed for such models are based on convex optimization and dual decomposition techniques.

Finally, a more complete and sophisticated model has been defined by Johansson and Xiao [13] in which they consider jointly the problems of rate optimization, packet routing and power allocation within the same mathematical framework. The resulting model fits into the class of nonlinear programs and the authors have proposed a column generation approach for its solution.

This paper could be considered as a further contribution in the direction of the work of Johansson and Xiao. We propose extensions both from the modeling and the algorithmic point of view. In particular, we explicitly consider for each source -destination pair a transmission demand and penalize in the objective function the failure to satisfy such demand. This penalty term can be considered as a QoS requirement for the network. Moreover, we define column generation procedures whicha re based on the solution of linear integer programming models, obtained by considering a suitable discretization of power and capacity levels. This is in contrast with the approach presented in [13], where non-linear programming models are considered. The advantage of our approach seems to be the ability of solving larger instances, without losing significance, since the introduced discretization appears to be quite natural in the application context.

In the rest of the paper, to facilitate the exposition, we will use the following convention: matrices will be denoted by upper case boldface characters while vectors will be denoted by lower case boldface characters. Moreover, if $\mathbf{A}$ is a matrix, we will use the notation $\mathbf{a}^i$ to refer to its i-th column and a_{ij} to refer to its (i, j)-th entry; likewise, if $\mathbf{x}$ is a vector, x_i will denote its i-th component.

The paper is structured as follows: in Section 2 the network model is introduced, Section 3 is devoted to the problem formulation, in Section 4 the column generation algorithms for capacity allocation and routing definition is developed, then our experimental results are reported in Section 5. Some concluding remarks are drawn in the last section.

2. Network Model

We consider the following scenario: a set of radio devices are scattered in the plane at fixed, known positions. Some of the devices need to send data to other devices, so we are given a set of source-destination pairs. For each source-destination pair we know the amount of data that must be transferred, on average, during one time unit (capacity demand). The source can communicate directly with the destination, if the latter is within the transmission range of the former. Otherwise, the data must be relayed in multiple steps (hops) through intermediate devices. We assume that the devices are equipped with sufficiently large memory buffers where data can be temporarily stored until having the chance to forward. The data is transmitted in packets, using a TDMA scheme. Hence, we assume that the time line is subdivided into equal intervals (time-slots) and that during each time-slot a non-idle device can either transmit or receive just one packet, whose length (number of bits) depends on the transmission rate. The power and transmission rate of a transmitting device can be changed at every time-slot and is chosen among a set of discrete values in order to guarantee the successful transmission of a data packet to the intended recipient, taking into account the interference from the simultaneous transmission of other devices. Our objective is to specify routing paths connecting all source-destination pairs and schedule the power and transmission rate on each time-slot for each device (zero power and rate meaning that the device is receiving or idle) in such a way that the average power is minimized, the lengths of the routing paths are minimized, and the average data flow along all the paths connecting each source-destination pair is at least equal to the capacity demand. Let us call "transmission scheme" the specification of the transmission power and rate of each device in a given time-slot. According to this approach, the overall optimization problem can be thought of as the combination of two different problems: the first one is to determine routing paths connecting the source-destination pairs and to select a set of transmission schemes to be used on the time-slots, specifying the relative frequency of use, in order to satisfy the demands. The second problem is to determine the actual scheduling of the transmission schemes on a slot by slot basis, using the selected schemes according to the specified frequencies. While any practical implementation, at the application level, of the proposed approach must consider also the second problem, the characteristic parameters that we have chosen to optimize, namely average transmission power, length of routing paths, and satisfaction of capacity demands, only depend on the solution of the first problem. Moreover, a feasible solution to the second problem can be easily determined by some heuristic procedure, given a solution to the first problem. For this reason, in the following we will focus on only upon the first problem, leaving the consideration of the second one to further investigations.

We model the network as a directed graph $G = (V, E)$ in which the node set V represents the set of devices and the edge set E represents the set of links (wireless connections) between the devices. For convenience a link will be represented as an ordered pair of nodes (u, v), with $u, v \in V$.

We denote by F the set of source-destination pairs and by τ^i, $(i = 1, \ldots, |F|)$ the set of directed paths in the graph connecting the i-th source-destination pair; we also denote by T the set of all the paths.

Therefore we introduce the *link-route incidence matrix* $\mathbf{R} \in \mathbb{R}^{|E|\times|T|}$ whose entries $r_{(u,v)t}$ are defined as

$$r_{(u,v)t} = \begin{cases} 1 & \text{if link } (u,v) \text{ lies in path } t \in T \\ 0 & \text{otherwise.} \end{cases} \tag{2.1}$$

For each $t \in T$, let x_t be the average data rate along path t; then the vector of average total traffic across the links is given by $\mathbf{Rx}$. Moreover, let χ_i $(i = 1, \ldots, |F|)$ be the average total flow between the i-th source-destination pair. We define the *paths matrix* $\mathbf{B} \in \mathbb{R}^{|F|\times|T|}$ whose entries b_{it} are:

$$b_{it} = \begin{cases} 1 & \text{if path } t \text{ belongs to set } \tau_i \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

Then, we can express $\boldsymbol{\chi}$ as follows:

$$\boldsymbol{\chi} = \mathbf{Bx}. \tag{2.3}$$

In addition, each source-destination pair i has a required minimum long-term data rate to be transmitted q_i which is an integral multiple of a suitable data rate unit λ. Since in general we cannot guarantee that all the data rate requirements can be satisfied, we associate with each source-destination pair i a deficiency term δ_i, that will be suitably penalized in the objective function, to take into account the possibility of unsatisfied demand. Then the demand constraint can be written as:

$$\boldsymbol{\chi} + \boldsymbol{\delta} \geq \mathbf{q}. \tag{2.4}$$

Taking into account interference and noise, each link (u, v) has a maximum sustainable long-term data rate (capacity) $\bar{c}_{(u,v)}$. Obviously, the actual data rate on each link must not exceed its capacity. This capacity constraint can be expressed as follows:

$$\mathbf{Rx} \leq \bar{\mathbf{c}}, \quad \mathbf{x} \geq 0. \tag{2.5}$$

As mentioned above, the actual data rate achievable at any link is affected by noise and interference. In particular, every link (u, v) is characterized by an instantaneous SINR (Signal to Interference and Noise Ratio) [1] value $\gamma_{(u,v)}$ defined as:

$$\gamma_{(u,v)} = \frac{g_{uv} p_u}{\sigma_v + \sum\limits_{\substack{u' \in V \\ u' \neq u,v}} g_{u'v} p_{u'}}, \tag{2.6}$$

in which, g_{uv} is the (known) power gain between the transmitter node u and the receiver node v. Hence, $\mathbf{G} \in \mathbb{R}_+^{|V| \times |V|}$ is the channel gain matrix of the network. Moreover, p_u is the instantaneous power used by the transmitter node u, and σ_v is the thermal noise power at the receiver node v.

Every link (u, v) can then be viewed as a single-user Gaussian channel whose Shannon instantaneous theoretical capacity is given by:

$$\tilde{c}_{(u,v)} = W \log_2(1 + \gamma_{(u,v)}) \tag{2.7}$$

where W is the system bandwidth. In the following we shall use formula (2.7) to express the SINR value at link (u, v) as a function of its instantaneous capacity:

$$\gamma_{(u,v)} = 2^{\frac{\tilde{c}_{(u,v)}}{W}} - 1. \tag{2.8}$$

[1] As customary, sometimes we express the SINR in dB units. We recall that $\gamma_{dB} = 10 \log_{10} \gamma$.

In order to attain the successful transmission of a data packet during a time-slot, the used transmission rate must not exceed the instantaneous theoretical capacity of the link. We propose to use the following simplifying hypothesis. We assume that the possible transmission rate and power levels used by a transmitting device are discretized and let $\{\epsilon^1, \epsilon^2, \dots, \epsilon^k\}$ be the discrete values of transmission rate (with $\epsilon^1 < \epsilon^2 < \cdots < \epsilon^k$) and let $\{\pi^1, \pi^2, \dots, \pi^h\}$ be the discrete values of the transmission power (with $\pi^1 < \pi^2 < \cdots < \pi^h$). Hence, during any given time-slot the transmission rate through link (u, v) can either be zero (if device u does not transmit to device v) or one of the allowable discrete values. Similarly, the transmission power used by a device u can either be zero (if the device is idle or receiving) or one of the allowable discrete values. For each value ϵ^i of transmission rate we can compute a corresponding minimum SINR value γ^i, compatible with such transmission rate (where $\gamma^1 < \gamma^2 <\ldots< \gamma^k$) by using formula (2.8) with $\tilde{c}_{(u,v)} = \epsilon^i$ and $\gamma_{(u,v)} = \gamma^i$. We use the computed values γ^i as threshold levels in the following sense: we assume that the transmission rate used on link (u, v) (instantaneous actual capacity) is ϵ^i if the SINR $\gamma_{(u,v)}$ at (u, v) is greater than or equal to γ^i and less than γ^{i+1} $(i = 1, \dots, k - 1)$. Moreover, we assume that the actual capacity on link (u, v) is zero if $\gamma_{(u,v)} < \gamma_1$ and ϵ^k if $\gamma_{(u,v)} \geq \gamma_k$. We introduce, for each link (u, v) and for each rate level i, a boolean variable $\omega^i_{(u,v)}$ whose value is one if and only if the actual capacity of link (u, v) is ϵ^i. The variables $\omega^i_{(u,v)}$ must satisfy the following constraints for each link (u, v):

$$\sum_{i=1}^{k} \omega^i_{(u,v)} \leq 1 \tag{2.9}$$

$$\gamma_{(u,v)} < \gamma^i \qquad \Rightarrow \omega^i_{(u,v)} = 0. \tag{2.10}$$

Therefore, for each $(u, v) \in E$, we can express its instantaneous actual capacity $c_{(u,v)}$ as follows:

$$c_{(u,v)} = \sum_{i=1}^{k} \epsilon^i \omega^i_{(u,v)}. \tag{2.11}$$

In addition, we introduce, for each device u and each power level j, a boolean variable φ^j_u whose value is one if and only if the instantaneous transmission power of u is π^j. Observe that if device u is idle or if it is receiving, then φ^j_u must be zero for any $j = i, \dots, h$. Variables φ_u^j must satisfy thus the following constraints for each node u:

$$\sum_{j=1}^{h} \varphi_u^j \leq 1 \tag{2.12}$$

$$\varphi_u^j = \begin{cases} 1 & \text{if the transmission power of } u \text{ is } \pi^{\mathrm{j}} \\ 0 & \text{otherwise.} \end{cases} \tag{2.13}$$

By using these new variables we can express p_u, for each $u \in V$, as follows:

$$p_u = \sum_{j=1}^{h} \pi^j \varphi_u^j, \quad \forall\, u \in V. \tag{2.14}$$

The transmission on link (u, v) can occur simultaneously to other transmissions only if its SINR $\gamma_{(u,v)}$ is greater than one of the thresholds γ^i. Thus the power and capacity feasibility constraint on link (u, v) can be expressed by the following inequality:

$$\frac{g_{uv} \sum_{j=1}^{h} \pi^j \varphi_u^j}{\sigma_v + \sum_{\substack{u' \in V \\ u' \neq u,v}} g_{u'v} \sum_{j=1}^{h} \pi^j \varphi_{u'}^j} \geq \sum_{i=1}^{k} \gamma^i \omega_{(u,v)}^i. \tag{2.15}$$

By exploiting the fact that, for any link (u, v), at most one of the variables $\omega_{(u,v)}^i$ is non-zero and letting e_i, for each i, denote a sufficiently large constant, the above non-linear inequality can be seen to be equivalent to the following linear system:

$$g_{uv} \sum_{j=1}^{h} \pi^j \varphi_u^j + e_i(1 - \omega_{(u,v)}^i) \geq \gamma^i \sigma_v + \gamma^i \sum_{\substack{u' \in V \\ u' \neq u,v}} g_{u'v} \sum_{j=1}^{h} \pi^j \varphi_{u'}^j \quad i = 1, \dots, k \tag{2.16}$$

A suitable value for the constant e_i is, for example:

$$e_i = \gamma^i (\sigma_v + \sum_{\substack{u' \in V \\ u' \neq u,v}} g_{u'v} \pi^h). \tag{2.17}$$

Furthermore, in order to simplify the notation, we introduce the following settings:

$$\begin{aligned} a_{(u,v)}^j &= g_{uv} \pi^j \\ f_{u'v}^{ij} &= \gamma^i g_{u'v} \pi^j \\ m_{(u,v)}^i &= \gamma^i \sigma_v - e_i \end{aligned} \tag{2.18}$$

Hence, we can rewrite (2.16) as:

$$\sum_{j=1}^{h} a^j_{(u,v)}\varphi^j_u - \sum_{\substack{u'\in V\\ u'\neq u,v}} \sum_{j=1}^{h} f^{ij}_{u'v}\varphi^j_{u'} - e_i\omega^i_{(u,v)} \geq m^i_{(u,v)}, \qquad \forall i=1,\dots,k, \quad \forall (u,v)\in E. \tag{2.19}$$

It is not difficult to show that the requirements that a link (u,v) is used only if the tail node u is transmitting, that no device transmits and receives at the same time and that no device can transmit or receive on different links simultaneously, can be modeled by the following constraints:

$$\sum_{j=1}^{h} \varphi^j_u \geq \sum_{i=1}^{k} \omega^i_{(u,v)}, \quad \forall (u,v)\in E \tag{2.20}$$

$$\sum_{j=1}^{h} \varphi^j_v + \sum_{i=1}^{k} \omega^i_{(u,v)} \leq 1, \quad \forall (u,v)\in E \tag{2.21}$$

$$\sum_{(u,v')\in E} \sum_{i=1}^{k} \omega^i_{(u,v')} + \sum_{(v',u)\in E} \sum_{i=1}^{k} \omega^i_{(v',u)} \leq 1, \qquad \forall u\in V. \tag{2.22}$$

Notice that, even though inequality (2.20) is implied by (2.19) when considering integrality, it is worth to retain it in the model because it is not implied in the linear relaxation. Notice also that inequalities (2.9) and (2.12) are dominated by inequalities (2.21), recalling that each node v has at least one incoming link $(u,v)\in E$. A power allocation scheme is given by a vector whose entries (one for each device) are equal to zero (if the corresponding device does not transmit) or equal to one of the discrete levels π^i. Let $\mathcal{P} = \{\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^L\}$ be the set of possible power allocation schemes; we can associate with $\mathcal{P}$ a suitable corresponding set of "actual capacity allocation schemes" $\mathcal{C} = \{\mathbf{c}^1, \mathbf{c}^2, \dots, \mathbf{c}^L\}$ such that the feasibility requirements are satisfied. By the above discussion, we can see that a feasible capacity vector $\mathbf{c}$ associated with a power allocation vector $\mathbf{p}$ can be determined by finding boolean vectors ω and φ such that (2.11), (2.14), (2.19), (2.20), (2.21) and (2.22) are satisfied. The long-term (average) power allocation vector $\bar{\mathbf{p}}$ and capacity allocation vector $\bar{\mathbf{c}}$ are weighted averages of the instantaneous power and capacity vectors, respectively, across all the time-slots of a suitably large time horizon. In other words, $\bar{\mathbf{p}}$ and $\bar{\mathbf{c}}$ lie in $conv(\mathcal{P})$ and $conv(\mathcal{C})$, respectively, where *conv* denotes the convex hull. Therefore, we can express $\bar{\mathbf{p}}$ and $\bar{\mathbf{c}}$ as follows:

$$\bar{\mathbf{p}} = \sum_{i=1}^{L} \alpha_i \mathbf{p}^i \qquad (2.23a)$$

$$\bar{\mathbf{c}} = \sum_{i=1}^{L} \alpha_i \mathbf{c}^i \qquad (2.23b)$$

$$\sum_{i=1}^{L} \alpha_i = 1, \quad \alpha_i \geq 0, \quad \forall i = 1, \ldots, L. \qquad (2.23c)$$

Moreover, the coefficient α_i associated with the power allocation scheme $\mathbf{p}^i$ (capacity allocation scheme $\mathbf{c}^i$) in the convex combination defining $\bar{\mathbf{p}}$ ($\bar{\mathbf{c}}$, respectively) is proportional to the number of time-slots of the time horizon in which the power allocation scheme $\mathbf{p}^i$ (capacity allocation scheme $\mathbf{c}^i$) is adopted. Let $\boldsymbol{\alpha}$ denote the vector of the convex combination coefficients, $\mathbf{P}$ the matrix whose columns are the vectors $\mathbf{p}^i$, and $\mathbf{C}$ the matrix whose columns are the vectors $\mathbf{c}^i$. Thus the long-term capacity and power vectors can be expressed in the following way:

$$\bar{\mathbf{p}} = \mathbf{P}\boldsymbol{\alpha}, \qquad (2.24a)$$

$$\bar{\mathbf{c}} = \mathbf{C}\boldsymbol{\alpha}, \qquad (2.24b)$$

$$\mathbf{1}^T\boldsymbol{\alpha} = 1, \quad \boldsymbol{\alpha} \geq 0 \qquad (2.24c)$$

where $\mathbf{1}$ denotes the vector whose entries are all equal to 1.

3. Problem Formulation

One of the objectives of our problem is the minimization of the cost incurred for the consumption of energy in the network due to transmission. In general, for each device, such cost would be proportional to the average transmission power, but the proportionality coefficient may be different for each device. If we let β_j be the unit power cost of the j-th device, then the total power cost of the network is given by: $\boldsymbol{\beta}^T\bar{\mathbf{p}}$.

Another goal is optimizing the routing of the packets in the network. The idea is to select for each source-destination pair a set of paths along which the packets are to be routed in such a way that the average "length" is minimized. We associate with each path t a "value" (or length) ϑ_t that shall be used in the objective function to weight the different data flows. The way in which the value ϑ_t is computed clearly depends on the metric we choose. For example, a natural choice would be the metric of "number of hops" given by:

$$\vartheta_t = \sum_{l=1}^{|E|} r_{(u,v)t}. \tag{3.1}$$

In this paper we propose a generalization of the "number of hops" metric, by assigning a weight $n_{(u,v)}$ to each link (u,v), and expressing $\boldsymbol{\vartheta}$ as

$$\boldsymbol{\vartheta} \;=\; \mathbf{n^T R}. \tag{3.2}$$

Hence the average length of the used paths is given by: $\boldsymbol{\vartheta}\mathbf{x}$.

A third objective we propose is QoS maximization, obtained by minimizing the loss of capacity with respect to a given capacity demand. We introduce the vector $\boldsymbol{\rho}$ whose components are associated with the source-destination pairs and represent a penalty cost that should be paid for each unit of unsatisfied demand.

Hence, we can consider the following linear programming formulation of the power and capacity allocation problem:

$$\begin{array}{rl}
\text{minimize} & \boldsymbol{\vartheta}^T\mathbf{x} + \boldsymbol{\beta}^T\bar{\mathbf{p}} + \boldsymbol{\rho}^{\boldsymbol{T}}\boldsymbol{\delta} \\
\text{subject to} & \boldsymbol{\chi} = \mathbf{Bx} \\
 & \boldsymbol{\chi} + \boldsymbol{\delta} \geq \mathbf{q} \\
 & \mathbf{Rx} \leq \bar{\mathbf{c}} \\
 & \bar{\mathbf{c}} = \mathbf{C}\boldsymbol{\alpha} \\
 & \bar{\mathbf{p}} = \mathbf{P}\boldsymbol{\alpha} \\
 & \mathbf{1}^T\boldsymbol{\alpha} = 1 \\
 & \mathbf{x} \geq 0 \\
 & \boldsymbol{\alpha} \geq 0 \\
 & \boldsymbol{\delta} \geq 0.
\end{array} \tag{3.3}$$

By easy simplification, we obtain an optimization problem in the variables $\mathbf{x}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$:

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{\vartheta}^T\mathbf{x} + \boldsymbol{\beta}^T\mathbf{P}\boldsymbol{\alpha} + \boldsymbol{\rho}^T\boldsymbol{\delta} \\ \text{subject to} \quad & \mathbf{B}\mathbf{x} + \boldsymbol{\delta} \geq \mathbf{q} \\ & \mathbf{C}\boldsymbol{\alpha} - \mathbf{R}\mathbf{x} \geq 0 \\ & \mathbf{1}^T\boldsymbol{\alpha} = 1 \\ & \mathbf{x} \geq 0 \\ & \boldsymbol{\alpha} \geq 0 \\ & \boldsymbol{\delta} \geq 0. \end{aligned} \tag{3.4}$$

We will refer to (3.4) as the *complete primal problem*. We shall also consider the dual of this problem, as follows. Let $\mathbf{w} \in \mathbb{R}^{|F|}$ be the dual variable vector corresponding to the primal constraints $\mathbf{B}\mathbf{x} + \boldsymbol{\delta} \geq \mathbf{q}$, let $\mathbf{y} \in \mathbb{R}^{|E|}$ be the dual variable vector corresponding to the primal constraints $\mathbf{C}\boldsymbol{\alpha} - \mathbf{R}\mathbf{x} \geq 0$ and let $z \in \mathbb{R}$ be the dual variable vector corresponding to the primal constraint $\mathbf{1}^T\boldsymbol{\alpha} = 1$. Hence, we have:

$$\begin{aligned} \text{maximize} \quad & z + \mathbf{q}^T\mathbf{w} \\ \text{subject to} \quad & \mathbf{B}^T\mathbf{w} - \mathbf{R}^T\mathbf{y} \leq \boldsymbol{\vartheta} \\ & \mathbf{C}^T\mathbf{y} + \mathbf{1z} \leq \mathbf{P}^T\boldsymbol{\beta} \\ & 0 \leq \mathbf{w} \leq \boldsymbol{\rho} \\ & \mathbf{y} \geq 0. \end{aligned} \tag{3.5}$$

We will call the above formulation the *complete dual problem*. The number of columns of matrices $\mathbf{B}$ and $\mathbf{R}$ (corresponding to the total number of paths between any source-destination pair) as well as the number of columns of matrices $\mathbf{P}$ and $\mathbf{C}$ (corresponding to the number of feasible allocation schemes) may be very large, so the generation of all such columns and the direct solution of the complete linear problems may be unfeasible. Instead, we propose the use of a classical column generation approach: we define suitable restricted primal and dual problems, solve them and evaluate the optimal dual solution. If such a solution does not violate the constraints of the complete dual problem, we stop: the solutions found are also optimal for the complete primal and dual problems. Otherwise we generate, by a suitable column generation algorithm, violated constraints of the dual problem (columns of the primal problem) to be added to the restricted problems and we repeat the procedure. Given the structure of the problem, we need two column generation procedures, to determine new feasible paths and new feasible power and capacity allocation schemes, respectively. We propose to define the initial restricted problems by considering the submatrices of $\mathbf{B}$ and $\mathbf{R}$ containing just one column for each source-destination pair, corresponding to the path of minimum length connecting the pair. We also propose that the initial submatrices of $\mathbf{P}$ and $\mathbf{C}$ contain just one column for each link: any such column corresponds to the trivial power and capacity allocation scheme in which only the associated link has a non-null capacity and the respective node, from which the link exits, transmits at the maximum power. In the following, we will denote by $\hat{\mathbf{B}}$, $\hat{\mathbf{R}}$, $\hat{\mathbf{P}}$, $\hat{\mathbf{C}}$, the submatrices of $\mathbf{B}$, $\mathbf{R}$, $\mathbf{P}$, $\mathbf{C}$, respectively, defining the restricted problems.

4. Column Generation Algorithms

Let $\hat{\mathbf{R}}$, $\hat{\mathbf{P}}$ and $\hat{\mathbf{C}}$ be submatrices of $\mathbf{R}$, $\mathbf{P}$ and $\mathbf{C}$ respectively and let $(\bar{\mathbf{x}}, \bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{\delta}})$ and $(\bar{\mathbf{w}}, \bar{\mathbf{y}}, \bar{z})$ be the optimal solutions for the restricted primal and dual problems, respectively. In general $(\bar{\mathbf{w}}, \bar{\mathbf{y}}, \bar{z})$ does not satisfy (3.5), that is, it violates some of the constraints. The goal of the column generation algorithm is to identify (generate) such violated constraints or, equivalently, the corresponding columns of the complete primal problem. Observe that we can generate two types of violated constraints: a constraint of the first type (type 1, in the following) corresponds to a new path for some source-destination pair (i.e., new columns for the $\hat{\mathbf{R}}$ and $\hat{\mathbf{B}}$ matrices); a constraint of the second type (type 2, in the following) corresponds to a new feasible power/capacity allocation scheme (i.e., new columns for the $\hat{\mathbf{P}}$ and $\hat{\mathbf{C}}$ matrices). In the following we describe the two corresponding column generation procedures.

4.1. **New Path Generation.** In order to find a new path to be considered for the routing, we look for a violated constraint of type 1. For this purpose, we solve, for each source-destination pair (s^i, d^i) (with $i = 1, \ldots, |F|$), the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & \bar{\mathbf{y}}^T \mathbf{r}^{(i)} + \vartheta_{\mathbf{r}^{(i)}} \\ \text{subject to} \quad & \text{“}\,\mathbf{r}^{(i)} \text{ incidence vector of a feasible path between } s^i \text{ and } d^i\,\text{”}. \end{aligned} \tag{4.1}$$

The optimal solution $\mathbf{r}^*$ of the above problem corresponds to a violated constraint of type 1 if

$$\bar{\mathbf{y}}^T \mathbf{r}^* - \bar{w}_i < -\vartheta_{r^*}. \tag{4.2}$$

For a given incidence vector $\mathbf{r}^{(i)}$, $\vartheta_{\mathbf{r}^{(i)}}$ is computed as:

$$\vartheta_{\mathbf{r}^{(i)}} = \sum_{l=1}^{|E|} n_{(u,v)} r^{(i)}_{(u,v)}. \tag{4.3}$$

Moreover the condition: “$\mathbf{r}^{(i)}$ is the incidence vector of a feasible path between s^i and d^i” can be expressed by means of a set of linear constraints. In conclusion the column generation problem can be formulated as the following linear programming problem:

$$\begin{aligned} \text{minimize} \quad & (\bar{\mathbf{y}}^T + \mathbf{n}^T)\mathbf{r}^{(i)} \\ \text{subject to} \quad & \sum_{(s^i,u)\in E} r^{(i)}_{(s^i,u)} = 1 \\ & \sum_{(u,d^i)\in E} r^{(i)}_{(u,d^i)} = 1 \\ & \sum_{(v,u)\in E} r^{(i)}_{(v,u)} - \sum_{(u,v)\in E} r^{(i)}_{(u,v)} = 0, \quad \forall\, v \in V - \left\{s^i, d^i\right\}. \end{aligned} \tag{4.4}$$

Observe that, in order to find the violated constraints of type 1 (if any), we only have to solve a small number of shortest path problems.

4.2. **New Power and Capacity Allocation Generation.** In order to find a new power/capacity allocation scheme we look for a violated constraint of type 2. To this purpose, we have to solve the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{\beta}^T \bar{\mathbf{p}} - \bar{\mathbf{y}}^T \bar{\mathbf{c}} \\ \text{subject to} \quad & \text{“}\bar{\mathbf{c}} \text{ feasible capacity allocation scheme”} \\ & \text{“}\bar{\mathbf{p}} \text{ the corresponding feasible power allocation scheme”}. \end{aligned} \tag{4.5}$$

The optimal solution $(\mathbf{c}^*, \mathbf{p}^*)$ of the above problem corresponds to a violated constraint of type 2 if

$$\boldsymbol{\beta}^T \mathbf{p}^* - \bar{\mathbf{y}}^T \mathbf{c}^* - \bar{z} < 0. \tag{4.6}$$

The feasibility conditions on the capacity and power allocation schemes are given by (2.11), (2.14),(2.19), (2.20), (2.21) and (2.22). Thus problem (4.5) can be modeled as the following {0, 1} linear programming problem:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{u \in V} \beta_u p_u - \sum_{(u,v)\in E} \bar{y}_{(u,v)} c_{(u,v)} \\
\text{subject to} \quad & \sum_{j=1}^{h} a^j_{(u,v)} \varphi^j_u - \sum_{\substack{u' \in V \\ u' \neq u,v}} \sum_{j=1}^{h} f^{ij}_{u'v} \varphi^j_{u'} - e_i \omega^i_{(u,v)} \geq m^i_{(u,v)}, \\
& \qquad\qquad \forall (u,v) \in E, \quad \forall i = 1, \dots, k \\
& c_{(u,v)} = \sum_{i=1}^{k} \epsilon^i \omega^i_{(u,v)}, \qquad \forall (u,v) \in E \\
& p_u = \sum_{j=1}^{h} \varphi^j_u \pi^j, \qquad \forall u \in V \\
& \sum_{j=1}^{h} \varphi^j_u \geq \sum_{i=1}^{k} \omega^i_{(u,v)}, \qquad \forall (u,v) \in E \\
& \sum_{j=1}^{h} \varphi^j_v + \sum_{i=1}^{k} \omega^i_{(u,v)} \leq 1, \qquad \forall (u,v) \in E \\
& \sum_{(u,v')\in E} \sum_{i=1}^{k} \omega^i_{(u,v')} + \sum_{(v',u)\in E} \sum_{i=1}^{k} \omega^i_{(v',u)} \leq 1, \\
& \qquad\qquad \forall u \in V \\
& \varphi^j_u \in \{0,1\}, \qquad \forall\, u \in V, \quad \forall j = 1, \dots, h \\
& \omega^i_{(u,v)} \in \{0,1\}, \qquad \forall\, (u,v) \in E, \quad \forall i = 1, \dots, k.
\end{aligned}
\tag{4.7}
$$

Here we note that, by a suitable trivial transformation, the model can be seen to have a very nice structure. First, we introduce some complementary variables and aggregated coefficients:

$$
\begin{aligned}
\bar{\varphi}^j_u &= 1 - \varphi^j_u \\
\bar{\omega}^j_u &= 1 - \omega^j_u \\
\bar{m}^i_{(u,v)} &= m^i_{(u,v)} - \sum_{\substack{u' \in V \\ u' \neq u,v}} \sum_{j=1}^{h} f^{ij}_{u'v} + e_i
\end{aligned}
\tag{4.8}
$$

Then, after some further simplification, we finally obtain the following formulation:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{u\in V}\beta_u\sum_{j=1}^{h}\pi^j\varphi_u^j+\sum_{(u,v)\in E}\bar{y}_{(u,v)}\sum_{i=1}^{k}\epsilon^i\bar{\omega}^i_{(u,v)} \\
\text{subject to} \quad & \sum_{j=1}^{\text{h}}a^j_{(u,v)}\varphi_u^j+\sum_{\substack{u'\in V\\ u'\neq u,v}}\sum_{j=1}^{h}f^{ij}_{u'v}\bar{\varphi}^j_{u'}+e_i\bar{\omega}^i_{(u,v)}\geq\bar{m}^i_{(u,v)}, \\
& \qquad\qquad\qquad\qquad\qquad \forall(u,v)\in E,\quad \forall i=1,\dots,k \\
& \sum_{j=1}^{h}\varphi_u^j+\sum_{i=1}^{k}\bar{\omega}^i_{(u,v)}\geq k, \qquad \forall(u,v)\in E \qquad\qquad (4.9)\\
& \sum_{j=1}^{h}\bar{\varphi}_v^j+\sum_{i=1}^{k}\bar{\omega}^i_{(u,v)}\geq h+k-1, \quad \forall(u,v)\in E \\
& \bar{\varphi}_u^j+\varphi_u^j=1, \qquad \forall\, u\in V,\quad \forall j=1,\dots,h \\
& \sum_{(u,v')\in E}\sum_{i=1}^{k}\bar{\omega}^i_{(u,v')}+\sum_{(v',u)\in E}\sum_{i=1}^{k}\bar{\omega}^i_{(u,v')}\geq \\
& |(u,v')\in E|+|\{(v',u)\in E\}|-1, \quad \forall u\in V \\
& \varphi_u^j\in\{0,1\}, \qquad \forall\, u\in V,\quad \forall j=1,\dots,h \\
& \bar{\omega}^i_{(u,v)}\in\{0,1\}, \qquad \forall\,(u,v)\in E,\quad \forall i=1,\dots,k.
\end{aligned}
$$

We can observe that in the above formulation all the variables are binary, all the constraint coefficients are non-negative and, with the only exception of the complementarity constraints $\bar{\varphi}_u^j+\varphi_u^j=1$, all the constraints are inequalities of the greater-than-or-equal type. In other words, if we relax the complementarity constraints, we get a multi-knapsack problem for which several preprocessing, formulation strengthening and exact or heuristic solution techniques are known in the literature. We have not exploited such a structure in the present work, but this line can be a possible direction for future work.

5. Computational Result

We have implemented the solution algorithm in C language and we have used CPLEX (version 9.1) as linear integer solver. All of our tests have been performed on a dual processor Opteron having a memory of 2 GB.

The networks used in our experiments have been generated by placing at random in a square area a set of nodes among which a pre-specified number of source-destination pairs has been selected. We have used a random generation procedure with uniform distribution. We have also specified a demand level of transmission capacity that must be achieved between each source-destination pair. Several experiments have been carried out by varying the number of nodes, the number of source-destination pairs, the demand level, the number of discrete levels of power and capacities and the size of the network area.

Initially the restricted problem is defined in the following way: for every source-destination pair, the shortest path is generated and the corresponding column is added to the $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{R}}$ submatrices. The initial transmission schemes correspond to the situations in which a single node on a single link transmits with maximum power; therefore the initial number of columns of the matrix $\tilde{\mathbf{P}}$ is equal to the number of links. The elements of $\tilde{\mathbf{C}}$ are consequently computed on the basis of these power allocations and amount to the number of links as well.

We have generated a large number of instances according to the following criteria. We have assumed that no transmission is possible (the capacity is equal to zero) over a link whose SINR is below a threshold of 10 dB; moreover, the thermal noise has been fixed to $3.34 \cdot 10^{-12}\ mW$.

For each link (u, v) the power gain g_{uv} has been computed by the following formula:

$$g_{uv} = \frac{\text{fad}_{uv}}{d(u,v)^3} \tag{5.1}$$

where fad_{uv} is a *fading parameter* randomly chosen between 10^{-5} and 10^{-3} and $d(u, v)$ is the euclidean distance between nodes u and v expressed in meters. We have also set:

$$g_{uv} = g_{vu}, \quad \forall (u,v) \in E. \tag{5.2}$$

We have assumed a bandwidth of 83.5 *MHz* (that corresponds to a channel capacity of $288, 86$ *Mbps* at the threshold level SINR of 10 dB) and considered five different levels of power (from a minimum of 0.5 mW to a maximum of 1000 mW) and ten levels of capacity (from a minimum of 290 *Mbps* to a maximum of 2000 *Mbps*).

The computational times to solve the problem range from a few seconds to several hours. Most of the running time is devoted to the task of determining new power and capacity allocation schemes. Moreover, the execution times are also influenced by the network size, by the distribution of the nodes and by the demand level. Therefore we have classified the instances according to the number of nodes, to the number of source-destination pairs, to the demand level and to the network size. For any class of problems, we have considered 10 instances and a time limit of one hour was given for solving any instance of each class. Our computational results, reported for several instances, have been shown in Tables $1-4$. The left hand side columns report those parameters characterizing the specific problem class, namely the number of nodes ($|V|$), the number of source-destination pairs ($|F|$) and the demand level (q_i), expressed in *Mbps*, for all source-destination pairs. In the right hand side columns $|E|$ indicates the average number of links, *Impr* represents the average percentage of the flow improvement obtained in the final solution with respect to the optimal flow in the initial restricted problem. With $|T|^*$ and $|A|^*$ we indicate the average number of paths and transmission schemes, respectively, added by the column generation procedures, P^* is the average value of total transmission power expressed in mW and *Sd* is the average percentage of the satisfied demand. The last two columns in the tables report the average execution time (T) expressed in seconds and the number of instances (*NS*) not solved within the time limit. All the averages have been calculated taking into account, over the 10 instances generated initially, only those really solved within the time limit.

Tables 1 and 2 report the results for instances having 10 nodes randomly distributed in a square area of $3km \times 3km$. In the first table we consider, for each size, three different demand levels (40 *Mbps*, 50 *Mbps* and 60 *Mbps*) whereas in the second one we vary the number of source-destination pairs (30, 40 and 50). Notice that some of the problems have not been solved within the time limit.

In Table 3 we are concerned with networks having 20 nodes randomly distributed in a square area of four different sizes: *A1* ($7km \times 7km$), *A2* ($10km \times 10km$), *A3* ($12km \times 12km$) and *A4* ($15km \times 15km$). While the area size increases, the execution time as well as the number of instances not solved within time limit

$\|V\|$	$\|F\|$	q_i	$\|E\|$	$Impr(\%)$	$\|T\|*$	$\|A\|*$	$P*$	$Sd(\%)$	T	NS
10	20	40	69,11	13,98	120,88	2,75	12,78	54,38	433,99	1
10	20	50	76,50	8,41	130,75	2,38	20,42	48,75	677,08	1
10	20	60	76,00	5,42	125,13	2,50	22,14	43,75	727,97	0

TABLE 1. Computational results by varying the demand

$\|V\|$	$\|F\|$	q_i	$\|E\|$	$Impr(\%)$	$\|T\|*$	$\|A\|*$	$P*$	$Sd(\%)$	T	NS
10	30	40	73,43	9,87	116,00	2,43	22,51	44,76	411,69	0
10	40	40	72,50	12,14	134,50	3,50	2,14	35,00	388,43	4
10	50	40	68,29	16,67	117,43	2,14	16,05	27,43	128,13	1

TABLE 2. Computational results by varying number of source-destination pairs

decreases. This fact can be explained by observing that with very low density networks, new transmission schemes become easier to find. Another observation concerns the percentage of demand satisfaction. As the area size becomes larger the *Sd* term decreases because, clearly, the nodes become more sparse and more transmission hops are required to get to destination. This factor, however, could be improved by increasing the penalty cost in the objective function but with the obvious consequence of a greater power consumption.

Finally, Table 4 reports the results concerning networks having a number of nodes ranging from 30 to 65 distributed in an area $A4$. Here we consider only two different levels of power, three capacity levels and 10 source-destination pairs. Besides we have fixed the time limit for instances of 65 nodes to three hours. As the number of nodes and the demand value increase, we can notice that it becomes more difficult to solve the instances within the time limit.

$\lvert V\rvert$	$\lvert F\rvert$	q_i	$Area$	$\lvert E\rvert$	$Impr(\%)$	$\lvert T\rvert*$	$\lvert A\rvert*$	$P*$	$Sd(\%)$	T	NS
20	20	20	*A1*	-	-	-	-	-	-	-	10
20	20	20	*A2*	66,67	10,29	39,33	1,11	55,08	45,00	354,57	0
20	20	20	*A3*	54,67	13,08	24,44	1,00	54,97	42,78	101,90	0
20	20	20	*A4*	53,00	12,86	20,20	1,20	18,09	40,50	37,29	0
20	30	20	*A1*	83,33	13,61	72,33	1,00	9,24	47,78	971,23	3
20	30	20	*A2*	63,50	10,28	46,38	1,00	18,90	34,17	431,67	0
20	30	20	*A3*	56,44	10,28	26,56	1,00	18,48	34,07	281,87	0
20	30	20	*A4*	54,80	9,80	27,50	1,30	20,46	34,33	42,12	0

Table 3. Computational results for different area sizes

$\lvert V\rvert$	q_i	$\lvert E\rvert$	$Impr(\%)$	$\lvert T\rvert*$	$\lvert A\rvert*$	$P*$	$Sd(\%)$	T	NS
30	10	88,60	6,52	22,00	0,80	10,35	79,00	13,98	0
50	10	178,00	10,01	52,44	0,89	6,42	61,11	587,74	1
65	5	298,57	6,45	98,29	1,14	11,96	100,00	1733,34	2
65	10	250,00	6,78	77,50	1,00	8,07	70,00	6991,64	8

Table 4. Computational results by varying number of nodes and demands

6. Conclusions and Future Work

In this paper we dealt with the routing and resource allocation problem for ad-hoc networks. We propose an optimization model that combines the routing definition and the power/capacity allocation within the same mathematical framework. With respect to previous works in the literature we included a routing definition approach that determines the optimal routing matrix for each problem data, rather than using a pre-established routing source-destination matrix; we use a discrete representation of the power (capacity) values that could be assigned to each node (link), and we consider an objective function that, besides minimizing of the number of hops, takes into account the power control and the QoS expressed as the level of demand satisfaction between each source-destination pair. To solve this problem we developed an exact algorithm based on a column generation approach. We avoid the typical non-linearity present in this kind of problems by discretizing the values of power (capacity) that could be assumed by each node (edge). We report some computational results that show the validity of our solution method on networks that have up to 65 nodes. As a comparison, the experiments reported in [13] deal with networks having less than 20 nodes, apparently because of the difficulties inherent to the non-linear formulations.

The results have shown that the method is very efficient for networks having a small number of nodes but for bigger networks the solution time increases rapidly. This is due, mainly, to the column generation procedure for finding new power and capacity schemes which represents, by far, the most time consuming part of the algorithm. Improving this task, hence, would mean not only to reduce the execution time but also to be able to solve problems with a greater number of nodes. This could be done, for example, by developing a specific method that takes into account the particular structure of the power and capacity column generation problem formulation. Indeed, we have already noticed that by relaxing some of the constraints the problem (4.9) becomes a variant of the multi-knapsack problem for which several techniques are available. The exploitation of such a structure will be one direction of investigation for future work.

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