The Stochastic Bid Generation Problem in Combinatorial Transportation Auctions

Chefi Triki[*,a,b], Simona Oprea[c], Patriza Beraldi[c], Teodor Gabriel Crainic[d,e]

[a] *Dept. of Mechanical and Industrial Engineering, Sultan Qaboos University, Muscat, Oman*
[b] *Dept. of Engineering for Innovation, University of Salento, Lecce, Italy*
[c] *Dept. of Electronics, Informatics and Systems, University of Calabria, Rende(CS), Italy*
[d] *Dept. Management et Technologie, École des Sciences de la Gestion, Montréal, Canada*
[e] *CIRRELT, Université du Québec à Montréal, Montréal, Canada*

Abstract

In this paper, we deal with the generation of bundles of loads to be submitted by carriers participating in combinatorial auctions in the context of long-haul full truckload transportation services. We develop a probabilistic optimization model that integrates the bid generation and pricing problems together with the routing of the carrier's fleet. We propose two heuristic procedures that enable us to solve models with up to 400 auctioned loads.

Key words: Auctions/bidding; Truckload procurement; Probabilistic constraints

1. Introduction

In a combinatorial auction (CA), the shipper (auctioneer) wants to procure long-haul full-truck transportation services in order to move a set of loads from their pickup to their delivery locations. Interested carriers have to submit bids to the auction on the transportation loads that they want to serve, not individually like in traditional auctions, but grouped into packages or bundles of loads.

CAs have been applied successfully in different application contexts. A non-exhaustive list of companies using CAs to procure transportation services includes Sears Logistics Services, The Home Depot, Walmart Stores, Compaq Computer Co., Staples Inc., The Limited Inc., Limited Logistics Services, Kmart Corporation, and Ford Motor Company [1, 2, 3, 4, 5, 6, 7].

Operations Research plays an important role in this context. Each carrier faces the *Bid Generation Problem (BGP)* in order to construct the bundles of loads to be submitted in the auction as bids. From the other side each auctioneer has to define the set of loads to be auctioned by solving the *Shipper Lane Selection Problem (SLSP)* and also has to solve the *Winner Determination Problem (WDP)* in order to allocate the loads to the winning carriers.

The BGP is a complex challenge that bidders, traditionally, avoid to face by submitting only a singleton load bids or by adopting simple strategies based on bidding on high value packages, on combining attractive

*Corresponding author
Email address: `chefi@squ.edu.om` (Chefi Triki)

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loads with less desirable loads in order to beat the competitors, or on putting together loads that increase its market share in a given area.

The complexity of the BGP derives from the necessity of evaluating an exponential number of potential bundles representing all the possible subsets that can be made out by the auctioned loads. It has been highlighted, indeed, in [8, 9] that the BGP is a NP-complete decisional problem. Another complicating aspect of the BGP consists in the necessity of taking into account the existent *synergies* between the loads. This is particularly important for the bidders when the items are characterized by complementarity, i.e., when the sum of the values of items taken singularly is smaller than the value of the bundle aggregating them (think, for example, about a pair of shoes that is worth more than the sum of the values of any unpaired shoe). This aspect of synergy plays a key role in the context of full-truckload (TL) long-haul transportation and such importance will be reflected in this contribution while developing our BGP model. Yet, relatively few contributions have targeted this important issue.

The present paper aims to address this challenging problem and has the goal of defining, designing, implementing and assessing innovative mathematical models and solution methods for the BGP in combinatorial auctions for the TL transportation procurement.

We consider here a single-round, one-sided, sealed-bid, first-price combinatorial auction in a transportation spot-market ([10]) and we develop an *advisor* that may assists carriers in their bidding decisions taking into account the dynamic planning of the transportation operations. Specifically, we try here to remedy the lack of available literature of integrated mathematical models that determine not only the most profitable bundle of loads to be submitted to the auction and its corresponding price, but also the resultant routing of the carrier's fleet in order to serve both the submitted bundle (a set of auctioned loads) and the pre-existent carrier's transportation commitments (called also booked loads in the sequel) in a stochastic settings. We propose, thus, a probabilistic mixed integer optimization model having the following innovative features:

- the consideration of the bundle price as a decision variable instead of computing it by using simplified formulas (as in [11, 12, 13]) or considering it a parameter equal to the sum of the ask prices of the bundle's loads (as in [14, 15]);
- the assessment of the stochastic nature of the problem by representing the auction clearing prices as random variables. Only Ergun et al. [16] dealt with this important aspect but for the simplified case of several single independent auctions running simultaneously instead of the CA considered in our model. Moreover, we model such clearing prices as normal random variables instead of the uniform ones as considered in [16];
- the inclusion into the model of a set of probabilistic constraints (chance constraints), which look up at guiding the selection of the bundle with the highest winning chance, i.e., the bundle whose price is guaranteed to be less than the corresponding clearing price by a certain probability threshold;
- the inclusion of the dynamic aspect of the routing problem by considering the space-time extended network for modelling the BGP (as only Chang did before in [14]);
- the determination of the routing of the trucks over the whole temporal horizon by solving a fleet management problem (and not a vehicle routing problem as proposed in [15] for the static variant of

the BGP);

- the enclosure of the time windows related to the pickup/delivery of every auctioned and booked load;
- the study of novel techniques for computing the synergies between the auctioned and booked loads and also within the bundle's loads.

The proposed model results to be, thus, more complete and more complex with respect to the already existing models for the BGP in combinatorial TL transportation auctions [17].

The combinatorial nature of the problem has limited the exact solution of our model only to instances of moderate size, i.e., up to a certain cardinality of the set of auctioned loads. For higher dimensions, heuristic procedures that permit a sequential solution of the BGP have been constructed. The behavior of the proposed solution approaches has been evaluated on a wide range of test problems randomly generated to represent realistic contexts.

The paper is organized in six sections. In Section 2 we review the main scientific literature. In Sections 3 and 4 we introduce our mathematical model and we present our solution approaches, respectively. In Section 5 we report our experimental experience and discuss the numerical results. Finally, we draw in Section 6 some concluding remarks.

2. Literature Review

The CAs have been proposed for the first time in 1982 by Rassenti et al. [18] for the solution of an airport time slot allocation problem. They are multi-item auctions in which the bidders can define their own combinations of items (called packages or bundles) instead of bidding on single items or predefined bundles. These are flexible but more complex trading techniques with respect to the simultaneous auctions [16] or the sequential auctions considered by Figliozzi et al. (see for example [19, 20]). Several excellent surveys have been published on how to design and run generic CAs [10, 21, 22, 23, 24].

In the TL transportation context, CAs offer significant advantages for both auctioneers and bidders. Auctioneers, indeed, can achieve more economic efficiency and transportation cost savings of up to 15 percent while maintaining or increasing their service levels [2]. Bidders can take into account their economies of scope while expressing their preferences on the packages to be served.

Despite these benefits, CAs involve many inherently difficult problems to be faced not only by the bidders but also by the auctioneers who must face the solution of both the SLSP [5] and the WDP [6]. While this last problem has been widely studied in the literature, there are only few studies that focused on the bid generation and evaluation problems. This scarcity of bibliographic contributions may be attributed to the objective difficulty of solving the BGP rather than its minor importance from the practical standpoint.

Crainic and Gendreau claim in [25] that carriers should analyze complex information available in the market and combine it with the management of their own fleet and personnel in order to determine their profitable bid strategies. The authors also point out that particular groups of loads may present a special interest for a given carrier due to the combinatorial nature of the carrier operations. Therefore, it is necessary to develop optimization-based decision support tools, the so-called *"advisors"*, to help carriers making their decisions by combining efficiently the market information, the planning and the operation procedures in order to evaluate and the select loads (see also [26, 27]).

Caplice and Sheffi argue in [28] that it is better to allow carriers to identify packages based on their own individual perspectives and networks since shipper specified packages result to be less successful for transportation. Caplice addresses in [29] several heuristic algorithms to help carriers generating open loop tours, closed loop tours, inbound/outbound reload packages, and short haul packages by using expected savings based on historical load volumes.

An et al. propose in [30] a model to assess the bundle values given the pairwise synergies for single-round, first price, sealed-bid forward combinatorial auctions. They also developed bundle construction algorithms for selecting profitable bundles. Their algorithms add as many profitable items as possible to a bundle given that the value of a bundle increases, on average, linearly with the bundle size. The synergy model proposed in [30] has as input the item values and pairwise synergy values and as output the bundle values for any combination. They also propose three bundling strategies focusing only on generating bundles (and not on pricing them) since they assume a fixed profit margin for their bids. However, since real data from CAs are generally not publicly available, their model has not been tested or validated.

Song and Regan propose in [13] a two-phase strategy to solve a TL vehicle routing problem (VRP) in order to generate bids. The first phase enumerates all the routes satisfying operational constraints to generate candidate bid packages. The second phase associates a binary variable with each candidate package, and solves a set partitioning problem to determine the desirable bids. Their approach simplifies the problem by assuming that all auctioned lanes (loads that must be serviced repetitively are usually called lanes) must be considered, and their objective is to minimize the total repositioning cost of empty trucks. Computationally efficient approximation methods for estimating the carrier's true values and for constructing bids have been also proposed.

In a previous contribution [12], Song and Regan solved the BGP in both scenarios, with and without presence of pre-existing commitments. Many limiting assumptions have been made such as the absence of a central depot and the unlimited carrier's capacity. Moreover, the carriers submit bids in a one-shot first-price reverse auction and do not take into account the competitors' behavior. Optimization-based approximation algorithms for solving the problem in each scenario have been also proposed.

Unlike [13], Lee et al. propose in [15] a VRP model that maximizes the profit deriving from generating the most profitable bundle instead of minimizing the total empty repositioning cost. Their optimization model integrates the simultaneous generation and selection of routes. Their work incorporates also pre-existing lane commitments and possible uncovered lanes together with capacity limits and other operational constraints. Given the prices for the lanes (assumed equal to the ask prices) they select only the best package to be submitted to the auctioneer. The model is a nonlinear integer program that has been solved by using a decomposition approach based on column generation and Lagrangian relaxation. Moreover, a decomposition heuristic based on a partitioning technique has been also developed.

Chang proposes in [14] a decision support model for the carriers participating in one-shot CAs. His bidding advisor integrates the load information in the e-marketplace with the carrier's fleet management plan and chooses hence the desirable bundle of loads. The bid generation and evaluation problems are formulated as a minimum cost flow problem that includes an approximation technique for estimating the average synergy values between loads.

Wang and Xia define in [32] the first-order synergy as the complementarity between a set of auctioned lanes and a set of booked lanes, and the second-order synergy as the complementarity between a pair of sets of auctioned lanes and booked lanes. The authors demonstrate that the synergy of a bid package may depend on other packages that could be won. In addition, they define the profit-based optimality criterion for a combinatorial bid, and based on some specific assumptions, they change the criterion into the cost-based optimality criterion. They take then the winning probability into account and show that the optimal solution of a VRP may lead to inferior bid packages. Their routing model assumes that all auctioned lanes must be serviced, and their objective is to minimize the total transportation cost. Their elaborate definitions of different synergies have not been, however, implemented in their BGP.

The analysis of the scientific literature clearly shows that the problem considered here has not been addressed before in an integrated way. In the present paper we try to fill this lack by proposing and solving a novel stochastic model.

3. The Optimization Model

The purpose of our study is to design a bidding advisor to help TL carriers in making bidding decisions in a CA for spot transportation markets. The TL carrier will apply such a bidding tool in order to generate the desirable bid and its associated price by solving an integrated bid generation and pricing problem, based on the currently known information within the planning horizon.

The known information to be used as input data for the model is:

- the number and location of the available vehicles at the beginning of the planning horizon;
- the booked and auctioned loads (their pickup and delivery locations, their associated pickup/delivery time windows);
- the ask price imposed by the shipper for each auctioned load.

The model solution will provide the most profitable bundle bid to be submitted together with its bidding price, represented in our problem as a decision variable. So far, among all the existing studies, only [16] considered the price of a bundle as a decision variable and not as a parameter (usually equal to the sum of the ask prices of the bundle's loads as suggested by the auctioneer).

We formulate the BGP as a time-space network-based fleet management problem that tightly integrates the information related to the auctioned loads with TL carrier's current transportation commitments [33, 34]. The proposed approach will generate only one bundle at a time, with its price and with the corresponding routing; i.e., the bundle that the carrier is confident to win with a given level of probability and that maximizes the carrier's profit. However, the carrier can clearly iterate the same decisional process in order to generate more bundles. While re-solving again the BGP the carrier can either exclude or not the bundle's loads already selected on the basis of the bidding language admitted by the auction (whether it is "OR" or "XOR"). If the auction rules do not allow overlapping bundles than the selected loads are excluded as new runs of the BGP are performed (as long as additional profitable bundles can be generated or till reaching a bidder-specified maximum number of bundles). If, however, the bidding rules do admit overlapping bundles than it is necessary, while re-running the BGP, to avoid re-generating the same bundle.

In the sequel we will introduce our probabilistic BGP, customizing it for the case of normal distribution of the auction clearing prices and also propose two ways for determining the synergies between the loads.

3.1. The BGP Formulation

Let $G = (V, A)$ be a directed complete graph, with V representing the set of cities and A the set of all possible links (roads) between the cities.

The BGP is formulated as a time-space extended network where the planning horizon of length T (one or more weeks) is divided into discrete intervals (days). For the formulation of our model the following notation is introduced:

Sets:

L_0 - set of carrier's pre-existing (booked) loads
L - set of loads being auctioned
B - set of all possible bundles of loads (power set of L), indexed by b
K - set of available trucks, indexed by k

Parameters:

Y_b - random variable denoting the lowest price offered by the competitors for bundle $b \in B$ (practically, the auction clearing price)
$\alpha \in [0, 1]$ - probability threshold
c_{ij} - cost of traversing arc $(i, j) \in A$
τ_{ij} - travel time for traversing arc $(i, j) \in A$ ($\tau_{ij} = \tau_{ji}$)
$R(L_0)$ - total revenue deriving from servicing the booked loads
d_i^k - binary indicator of trucks position equals to 1 if truck k is in city i at $t = 1$, and 0 otherwise
$i(l)/j(l)$ - pickup/delivery cities of load $l \in L \cup L_0$
$a(l)/b(l)$ time window extremes (pickup feasible time) for picking up load $l \in L \cup L_0$

Decision variables:

y_{ij}^{kt}: binary variable equals to 1 if truck k is moving from city i to city j starting at time t, 0 otherwise
x_b: binary variable taking value 1 if bundle b is chosen to be submitted, 0 otherwise.
p_b: bidding price for bundle b

The following formulation for the BGP considers the pickup time-windows for the auctioned and booked loads. A similar model can be proposed by taking into account instead the delivery time-windows.

$$\max \sum_{b \in B} p_b x_b + R(L_0) - \sum_{k \in K} \sum_{(i,j) \in A} \sum_{t=1}^{T-\tau_{ij}} c_{ij} y_{ij}^{kt} \tag{1}$$

s.t.

$$P(p_b x_b \leq Y_b) \geq 1 - \alpha \qquad \forall b \in B \tag{2}$$

$$\sum_{b \in B} x_b \leq 1 \tag{3}$$

$$\sum_{j \in V} y_{ij}^{k1} = d_i^k \qquad \forall i \in V, \qquad \forall k \in K \tag{4}$$

$$\sum_{j \in V : t+\tau_{ij} < T} y_{ij}^{kt} = \sum_{j \in V : t-\tau_{ji} \geq 1} y_{ji}^{k(t-\tau_{ji})} \quad \forall i \in V, \qquad \forall t \geq 1, \qquad \forall k \in K \tag{5}$$

$$\sum_{k \in K} \sum_{t \in [a(l), b(l)]} y_{i(l)j(l)}^{kt} \geq 1 \qquad \forall l \in L_0 \tag{6}$$

$$\sum_{k \in K} \sum_{t \in [a(l), b(l)]} y_{i(l)j(l)}^{kt} \geq x_b \qquad \forall b \in B, \qquad \forall l \in b \setminus L_0 \tag{7}$$

$$\sum_{k \in K} \sum_{t \in [a(l), b(l)]} y_{i(l)j(l)}^{kt} \geq 2x_b \qquad \forall b \in B, \qquad \forall l \in b \cap L_0 \tag{8}$$

$$y_{ij}^{kt} \in \{0, 1\} \qquad \forall i \in V, \qquad \forall j \in V, \qquad \forall k \in K, \qquad \forall t \geq 1 \tag{9}$$

$$x_b \in \{0, 1\} \qquad \forall b \in B \tag{10}$$

$$p_b \geq 0 \qquad \forall b \in B \tag{11}$$

The objective function (1) maximizes the profit defined as the difference between revenues (deriving from servicing both booked and auctioned loads) and total cost of the routing. The probabilistic constraints (2) impose a minimum reliability for a bid to be submitted and to result successful. Constraint (3) will force the model to choose at most one bundle, the most convenient one (and, as noted above, one can solve again the same model if more bundles are needed). Constraints (4) give information on the location of each available truck at the beginning of the planning horizon. Equations (5) are the flow balance constraints and constraints (6) will force the model to serve all the booked loads within their pre-specified time windows. Constraints (7) represent the logical relationship between the routing variables and the bids selection variables; only the auctioned loads that belong to the bundle should be served within their time windows. Moreover, if it happens that a booked load and an auctioned load have the same pickup and delivery cities and the same time windows then constraints (8) will ensure to serve both of them. Finally, constraints (9)–(11) are the domain definitions for the decision variables.

It is worth noting that, even though the above BGP model results to be nonlinear (because of both (1) and (2)), this does not represent any computational challenge because it can be easily linearized.

A distinctive feature of the proposed model is the assessment of the stochastic nature of the bidding process by the definition of probabilistic constraints on the bidding price. It expresses the degree of certainty of submitting a winning bid or, in other words, the risk that the decision maker is willing to assume in submitting a bid that can result unsuccessful. For example, if α is 0.05 then constraints (2) will ensure that "in 95% of the cases the bid price for bundle b is lower than its clearing price" or "there is a 95% chance that the submitted bundle will be successful in the auction" or also "the bidder, while submitting bid b, can tolerate a risk of 5% that it will not result successful".

Such probabilistic constraints (2), introduced above for a generic random variable, will be expressed now according to the normal distribution of the clearing prices.

3.2. Normal Distribution for the Probabilistic Constraints

In order to incorporate the uncertainty related to the BGP, the auction clearing price (i.e., the winning price) for each load is modeled as a continuous random variable. In order to develop the deterministic equivalent form of constraints (2) we need to assume a specific distribution of each load clearing price. Contrarily to Ergun et al. who assumed in [16] a uniform distribution, we assume here a normal distribution of the clearing prices. Our choice is based on the fact that the prices usually follow in practice a Gaussian trend and because the properties of this kind of distribution are more suitable to the problem under exam (with respect to the Weibull or Gumbel ones). Clearly, the optimization model continues to be valid by using any other distribution that suits the application context, possibly at the cost of further burdensome computations.

The question to be discussed here is how to combine the individual clearing prices of a bundle loads in order to compute the bundle clearing price Y_b.

Consider a bundle b consisting of n auctioned loads to bid on and denote by X_l the random variable representing the clearing price of every load $l = 1, .., n$. We assume here that each X_l follows a normal distribution with mean μ_l and variance σ_l^2:

$$X_l \equiv N(\mu_l, \sigma_l^2).$$

Note that the mean μ_l and variance σ_l^2 values can be determined by means of forecasting techniques or by collecting samples of the shipping fares that several competing carriers make available through either their websites (as a matrix form referring to many origin-destination pairs) or their call centers. These fares may represent the observations on the basis of which the probability distribution is approximated and the mean and variance values are estimated.

In practice, the random variables are dependent because the bid price of each load depends on its synergy with any other load. For example, the bidding price of a bundle A involving two loads, say Milan–Rome and Rome–Milan, that could be combined in a round trip, would be less than the bidding price of a bundle B composed of the separate loads Milan–Rome and again Milan–Rome. Bundle A, indeed, exhibits a strong synergy between its two loads, whereas the loads of bundle B are completely independent. The synergy is, thus, the tool that the bidder can use to ensure the economies of scope within his transportation network. It represents, as widely discussed for example in [2, 30], a key issue in optimizing the selection of any transportation commitment.

However, a rigorous study of the synergy would involve the modeling and the simulation of multivariate dependent random variables, which is a complex task that goes beyond the goal of this paper. Rather, we assume here the random variables X_l ($l \in b$) to be independent and normally distributed but we introduce a corrective term in order to express the synergy among bundle's b loads.

We model thus the random variable Y_b representing the clearing price of bundle b, as follows:

$$Y_b = S_b \, . \, \sum_{l=1}^{n} X_l,$$

where S_b is the parameter quantifying the synergy level within bundle b. The way of computing S_b will be discussed in the next subsection.

By using the invariance property of the normal distribution with respect to the sum of independent random variables, it follows that $Y_b \equiv N(\mu_b, \sigma_b^2)$ where:

$$\mu_b = S_b \,.\, \sum_{l=1}^{n} \mu_l \qquad \text{and} \qquad \sigma_b^2 = S_b^2 \,.\, \sum_{l=1}^{n} \sigma_l^2$$

and hence:

$$Y_b \equiv N(S_b \,.\, M_b \,,\, S_b^2 \,.\, n)$$

where $M_b = \sum_{l=1}^{n} M_l$.

The chance constraints (2) can be expressed as following:

$$p_b x_b \leq S_b[\sum_l \mu_l + \Phi^{-1}(\alpha) S_b \sum_l \sigma_l^2] \quad \forall b \in B,$$

where Φ^{-1} is the inverse function of the cumulative distribution function for a standard normal distribution.

3.3. Bundle Synergy Computation

The computation of the synergy between loads is very often ignored in the literature related to generating and pricing bids for transportation auctions. To the best of our knowledge, only few contributions have considered the synergy in developing and solving BGP models ([14, 15, 32]) and the only study that explicitly shows how to compute the synergies within a bundle of loads is due to An et al. [30]. This last paper proposed a model for evaluating the value of a bundle, given the pairwise synergy values, and for assessing the performance of different bundling strategies under deterministic settings.

Given the settings of our problem and its probabilistic nature, we need to develop a different approach for incorporating the synergies into the model. We have included, indeed, the synergies in the bundle's clearing price expression because of the interaction existing, from one side mutually between the bundle's auctioned loads and from the other side between the booked and the auctioned loads.

It is very difficult for a bidder to define a closed-form formula for computing the synergies. Therefore we propose in the sequel two different ways for approximating the pairwise synergy values that will be then used to calculate the synergy of bundle b, as follows:

$$S_b = \frac{\sum_{j=1}^{|b|} \left(\sum_{l=1}^{|L_0|} pairwisesynergy(j,l) + \sum_{k<j} pairwisesynergy(j,k) \right)}{|b||L_0| + \binom{|b|}{2}}.$$

The above expression of S_b is nothing but the average of the synergies over all pairs of loads within bundle b and also those between the booked and bundle b loads.

3.3.1. Method 1: Distance-based Pairwise Synergy

Our first approach for computing the loads' pairwise synergy is based on the analysis of the distance between loads defined, for simplicity, as the travelling time between the origin of a load and the destination of its predecessor load.

If two loads result to be time-incompatible this means that there is no complementarity between them, and the synergy value is, thus, set to 1. If the loads result instead to be time-compatible then they will have a pairwise synergy value that depends on their distance and on the number of overlapping days of their time-windows. Specifically, the pairwise synergy improves when the two loads have many common days in their time windows or as their distance decreases.

3.3.2. Method 2: Hop-based Pairwise Synergy

Our second approach is based on the concept of hop defined as the number of arcs that a vehicle has to travel unloaded for a repositioning action in order to reach a pickup node of another load. Since the vehicle travels unloaded, a hop represents for the carrier only a loss corresponding to the travelling cost without any revenue. Therefore, the number of hops will represent here another measure of the synergy level; the lower is the number of hops in the routing plan, the better is the synergy and, consequently, smaller will be the cost supported by the carrier.

Given two generic nodes, one of the following situations can happen:

- no empty repositioning is necessary: *number of hop*$= 0$;
- one direct arc for repositioning is required: *number of hop*$= 1$;
- m intermediate nodes should be traversed for repositioning: *number of hop*$= m + 1$.

In these last two cases, while determining the number of hops, we should also check which among the involved arcs correspond to a booked or auctioned load. If this happens and moreover time compatibility is ensured then we have to reduce the corresponding number of hops. Specifically, assume, for example, that a direct arc exists between two nodes, that is *number of hop*$= 1$. If this arc corresponds to an auctioned or to a booked load it will not be considered as a hop, and hence, the number of hops will be decreased by 1. If, however, the same arc results to be time incompatible with the other loads, then the number of hops remains unchanged.

4. The Heuristic Solution Approaches

The combinatorial nature of the proposed model prevents the exact solution of the problem to only instances of moderate sizes. Preliminary results have shown, indeed, that, by using the CPLEX Branch-and-Cut method, even problems with 20 auctioned loads can be hardly solved within an acceptable amount of time. For higher size of the BGP model, heuristic solution techniques should be developed.

In the sequel, we propose two heuristic solution methods having both a number of bundles evaluation of the order of $|L|^2$ (i.e., having $\mathcal{O}(|L|^2)$ as complexity).

4.1. Heuristic I: Sequential Descending Method

This first heuristic method starts with the maximal cardinality bundle (i.e., the whole set of auctioned loads L), evaluates the incremental profit $\rho(l)$ (marginal benefit) that may be obtained by dropping out one load, say l, at a time and chooses the one yielding the best incremental profit. The current bundle is then updated, by eliminating the selected load to get set $L^{'}$, and the process is repeated till reaching the empty set. The best feasible solution of the BGP will correspond, thus, to the time-compatible bundle including all the loads l^* satisfying the condition $\rho(l^*) \leq 0$; If we denote by $f(L^{'})$ the objective function value obtained

```
Start with L' := L as initial subset
1 :  FOR ALL l ∈ L'
compute the marginal benefit of dropping load l from L' as:
ρ(l) := f(L' \ {l}) − f(L')
Determine l* := argmax ρ(l)
Update L' as L' := L' \ {l*}
IF L' is empty THEN STOP, the solution is L'
including all loads l* satisfying ρ(l*) ≤ 0
ELSE GO TO 1.
```

Figure 1: Sketch of the sequential descending heuristic

by solving the BGP (1)–(11) when the set of bundles of auctioned loads is reduced to $L^{'} \subseteq L$, then Heuristic I can be described as sketched in Figure 1.

If, during the iterative process, it happens that $argmax\ \rho(l)$ does not correspond to a single load but to a set of elements, then l^{*} can be chosen from this set on the basis of any logical criterion. Possible choices include, for example, selecting the bundle $b = L^{'} \setminus \{l^{\star}\}$ having the best synergy value S_b or the one corresponding to the highest ask price M_b.

Proposition 4.1. *T*he number of bundle evaluations in the sequential descending heuristic is $\frac{n(n+1)}{2}$ instead of $2^n - 1$, the cardinality of the whole set of possible bundles.

Proof. Heuristic I evaluates first the bundle corresponding to $L^{'} := L$ then n bundles of cardinality $n-1$ each, then the possible $n-1$ bundles having cardinality $n-2$ each, and so on until $L^{'}$ becomes empty. Hence there are all together $\sum_{i=1}^{n} i$ bundles evaluated, that is $\frac{n(n+1)}{2}$ evaluations.

The sequential descending heuristic can be explained in terms of graph representation. The possible subsets (bundles) of L of cardinality n can be thought, indeed, as the nodes of a tree. The root node (level 0 of the tree) represents set L (of maximal cardinality) and the nodes of level i ($i = 1, \ldots, n-1$) denote the subsets of cardinality $n - i$. The number of nodes at level i is given by the binomial coefficient $\binom{n}{i}$ which is, by the symmetry property, equal to $\binom{n}{n-i}$. Applying heuristic I means, thus, exploring in depth the whole tree by choosing at each level only one node till reaching the leaf corresponding to the empty bundle; The solution will then correspond to the bundle including all the loads characterized by a negative marginal benefit.

4.2. Heuristic II: Sequential Ascending Method

A variant of the previous heuristic can be obtained by considering as the initial bundle the one with minimal cardinality (for example, the empty set) and evaluating the incremental profit deriving from adding a load at a time to the current bundle. The iterative steps of heuristic II are summarized in Figure 2. The key difference with respect to heuristic I is that we have introduced here a different exit criterion that will force the process to be halted as soon as no positive marginal profit can be achieved, without reaching necessarily the full cardinality bundle. The bid to be submitted will be defined, in this case, as the time-compatible

Start with $L^{'} := \emptyset$ as initial subset

1 : FOR ALL $l \in L \setminus L^{'}$

compute the marginal benefit of adding load l to $L^{'}$ as:

$\rho(l) := f(L^{'} \cup \{l\}) - f(L^{'})$

Determine $l^{\star} := argmax\ \rho(l)$

IF $\rho(l^{\star}) > 0$ THEN

update $L^{'}$ as $L^{'} := L^{'} \cup \{l^{\star}\}$

IF $L^{'} = L$ OR $\rho(l^{\star}) \leq 0$ THEN STOP, ELSE GO TO 1.

Figure 2: Sketch of the sequential ascending heuristic

bundle $L^{'}$ obtained after adding load l^{*} corresponding to the last load satisfying $\rho(l^{*}) > 0$, and its value will be $f(L^{'}U\{l^{*}\})$.

Even in this case, we can explain analogously the sequential ascending heuristic in terms of a tree decision problem, as previously described. The root of the tree, in this case, will represent the empty set and the nodes of level i ($i = 1, \ldots, n$) will denote the different subsets of cardinality i. Even heuristic II performs an exploration in depth of the tree but exits whenever no marginal profit can be ensured without necessarily reaching the last leaf corresponding to the bundle including all the auctioned loads.

Proposition 4.2. *T*he number of bundle evaluations in the sequential ascending heuristic is at most $\frac{n(n+1)}{2} + 1$ instead of $2^n - 1$.

Proof. The proof can be easily derived by following the same arguments of the proof of proposition 4.1.

5. Computational Experiments

Several computational experiments have been carried out in order to validate the developed BGP model and to measure the efficiency of the proposed heuristics. Since in general no real data is publicly available for CAs (as often reported in the literature, for example in [30]), a test problem generator has been specifically constructed and implemented in this paper.

The experiments have been performed on a computer with an Intel Xeon X5680 processor having 3.33 GHz and 47 GB of RAM. The model has been implemented and compiled by using version 6.0 of Microsoft Visual C++ integrated with IBM ILOG CPLEX (version 12.2.0) for solving all the arising MIP models. The ILOG CONCERT Technology was also used.

5.1. Test Problems Configuration

In order to simulate CAs data, an automated problems generator has been developed obtaining, thus, a significant set of test instances.

We considered a planning horizon of 1 week having 7 working days and we created the complete directed graph $G = (V, A)$ as defined in section 3.1. We then calculated, for the resulting graph, the shortest paths (i.e., minimum distance between each pair of nodes) by using the Floyd Warshall's algorithm. The sets L_0

of booked loads and L of auctioned loads will be generated by selecting randomly two subsets (of equal cardinality) among the graph's arcs.

The travelling time matrix τ, the distance matrix D and the cost matrix c have been generated as follows:

- Matrix τ has been randomly generated according to a parameter uniformly distributed in the range $[1, 3]$; we have assumed that $\tau_{ii} = 1$ for every $i \in V$ and $\tau_{ij} = \tau_{ji}$ for every $i, j \in V$ (symmetric matrix);
- The elements of matrix D are considered to be proportional to τ_{ij} taking into account the average speed of the trucks ($90\,km/h$) and the number of driver's working hours ($8\,h/day$); We also set $D(i,i) = 0$;
- The value of each element of c is directly proportional to the distance, defined by the following formula:

$$c_{ij} = [270 + random(0, 50)] \cdot D(i, j),$$

  where the value 270 (Euros) represents, for arc (i, j), the daily fixed cost (fuel and driver's wage) and $random(0, 50)$ is a random number uniformly distributed in $(0, 50)$ representing possible cost variations (in the fuel price, highway tolls, etc.).

A time window was associated to every booked and auctioned load. In both cases, the window extremes represent the first and the last possible day for picking up the load. The first extreme $a(l)$ associated with load $l \in L_0 \cup L$ was randomly generated according to the formula: $a(l) = uniform(1, T - \tau_{ij})$ and the second extreme $b(l)$ was obtained by the same formula and by imposing, additionally, that $b(l) \geq a(l)$.

The number of trucks is chosen based on the practical information that the typical daily ratio of loads per vehicle ranges from 2 to 2.5 [35]. Next, if the number of booked loads to be picked up on the first day of the time horizon exceeds the number of available trucks then the latter will be increased. The trucks' departure positions have been assigned randomly to the nodes but we have checked, though, that the generated problems are meaningful by performing a preprocessing analysis.

The ask price M_b associated to a bundle b is computed by increasing the sum of the transportation costs of each load in the bundle by 60%, whereas the revenue derived from each booked load is set to be 40% more than its cost. The probabilistic threshold α has been set to 0.05 along all the experiments.

The pairwise synergy values associated to each pair of loads is produced on the basis of the number of overlapping days in the delivery time windows and either the distances or the number of hops between the loads, as described in section 3.3. Ten levels have been chosen to characterize the synergy defined as follows: if two loads are time-incompatible their synergy will be 1; If temporal compatibility applies then a synergy value ranging from 0.95 to 0.50 is assigned. For implementation convenience, the former corresponds to a scarce interaction between the loads, while the latter represents the maximum possible pairwise complementarity. The synergy values used in our experiments are reported in Table 1.

5.2. Numerical Results

Since no benchmark test problems exist, we start by assessing the performance of our heuristics with respect to the exact solution. As mentioned above, solving exactly our probabilistic BGP was possible only for a limited number of auctioned loads. Table 2 reports the results obtained as the average of 8 problem instances when fixing the number of auctioned loads to 10 and varying the number of nodes. It includes, for

		distance				hop		
		0	1	2	3	0	1	2
Overlapping days in the time windows	**0**	0.6	0.8	0.95	0.95	0.6	0.8	0.95
	1	0.55	0.75	0.90	0.90	0.55	0.75	0.90
	2	0.55	0.75	0.90	0.90	0.55	0.75	0.90
	3	0.5	0.7	0.85	0.85	0.5	0.7	0.85
	4	0.5	0.7	0.85	0.85	0.5	0.7	0.85
	5	0.5	0.7	0.85	0.85	0.5	0.7	0.85

Table 1: Distance-based and hop-based pairwise synergy values

an increasing number of cities (nodes), the CPU time for solving the BGP model by the exact solver and by the proposed heuristics. It also reports for both heuristics the time reduction with respect to the exact solution time.

These results show clearly significant improvements in terms of execution times for both heuristics with respect to the exact solution, reaching an average time reduction of more than 80% for heuristic I and about 90% for heuristic II. When the solution quality is considered, we have observed that while heuristic II succeeds to find the same solution obtained by the exact solver for all the test instances, heuristic I fails in most cases generating an average gap of 45.7% with respect to the profit obtained by the exact solution.

	Exact	**Heuristic I**		**Heuristic II**	
Number of cities	**Time(s)**	**Time(s)**	**Reduction(%)**	**Time(s)**	**Reduction(%)**
10	12.584	3.254	75	2.261	82
15	60.812	7.470	88	2.970	95
20	71.289	12.768	82	6.889	90

Table 2: Exact vs. heuristics average CPU time comparison

For higher dimensions, our attention has been devoted to a direct comparison of the performance of both heuristics. We want to check if the trend of superiority of heuristic II with respect to heuristic I observed in Table 2, though for small problems, will continue to be valid for large-scale problems.

Our experimental results in this direction are summarized in Table 3 where problems with up to 400 auctioned loads have been solved. We report in the first column the test problem characterized by the triple attributes: number of cities−number of auctioned loads−number of trucks. Then we report, for both heuristics, the value of the objective function ($OF(b^*)$) and the cardinality ($|b^*|$) corresponding to the submitted bundle (b^*) and also the CPU solution time. Finally, we include the relative improvement achieved by heuristic II over heuristic I in terms of solution values (OF-Gap) and CPU solution time (T-Gap). All the results are averaged over 5 instances generated for each test problem. When it happens for some instances to be unfeasible (mainly because of the insufficient number of trucks) the averages were calculated over the solved instances only.

From the results reported in Table 3 we notice that the trend previously observed in the case of small

Problem	Heuristic	OF(b^*)	$\lvert b^* \rvert$	Time(s)	OF-Gap(%)	T-Gap(%)
12-100-40	H2	19694.1	7	243.3	1.27	16.64
	H1	19442.4	7	291.9		
13-125-50	H2	26336.9	10	759.3	0	6.39
	H1	26336.9	10	811.2		
15-150-60	H2	29640.9	6	1548.9	0.69	14.79
	H1	29436.4	6	1817.4		
18-200-80	H2	36577.2	8	3260.7	0	42.39
	H1	36577.2	8	5616.7		
20-250-100	H2	46851.9	7	6783.4	1.31	45.04
	H1	46242.5	6	12343.7		
21-300-120	H2	63288.4	6	18478.4	0.44	33.19
	H1	63007.6	5	24611.4		
23-350-140	H2	63439.5	6	26807.3	0.29	66.41
	H1	63249.2	7	44611.4		
24-400-160	H2	71391.6	11	179131.0	0	8.18
	H1	71391.6	11	193800.0		

Table 3: Comparison between heuristic I (H1) and heuristic II (H2) for large-scale problems

problems has been confirmed even for larger problems and heuristic II clearly still outperforms heuristic I in all the test problems. The objective function values produced by heuristic II are, indeed, in general greater than those obtained by heuristic I even when the cardinality of the submitted bundle is smaller and also the CPU times are constantly lower. Concerning the quality of bundles, the two heuristics produce exactly the same set of bundles in the case of three test problems (characterized by a zero value of the OF-Gap), and even in the other test problems similar bundles (and consequently similar bidding prices) have been generated. We can conclude, thus, that the real advantage of heuristic II over heuristic I can be expressed in terms of CPU time reduction rather than in terms of the objective function increase.

6. Conclusions and Future Research

This paper has represented a thorough study and an accurate analysis of the BGP in a combinatorial auction for full truckload transportation procurement. The major scientific contribution consisted in developing a probabilistic mathematical model that integrates the bid generation and pricing problems together with the routing of the carrier's fleet. Two heuristic procedures that enable solving complex instances with up to 400 auctioned loads have been also proposed.

Extensive computational tests have been conducted in order to evaluate, first, the performance of the proposed heuristics with respect to the exact solution and then to understand which heuristic performs better. The results have shown that the heuristic that starts with an empty bundle and then includes sequentially the most profitable loads outperforms its counterpart that starts with the full cardinality bundle and drops sequentially non profitable loads.

While we think that this work has faced an interesting research topic in the context of BGP, we are conscious that it opens up several interesting research points along different directions. First, novel preprocessing procedures of the auctioned loads would be developed in order to reduce a priori the exponential number of bundles and overcome the curse of dimensionality, with the ultimate goal of being able to solve bigger realistic problems. Second, more sophisticated and effective ways for computing the bundle synergy would be of great benefit to the solution of BGP. In this same direction, the dependence among the random variables characterizing the clearing prices of the bundle loads should be investigated. Finally, our proposed BGP model defined specifically for one-shot combinatorial transportation auctions is a valid basis for developing novel models for multi-round auctions.

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